 Name: Key

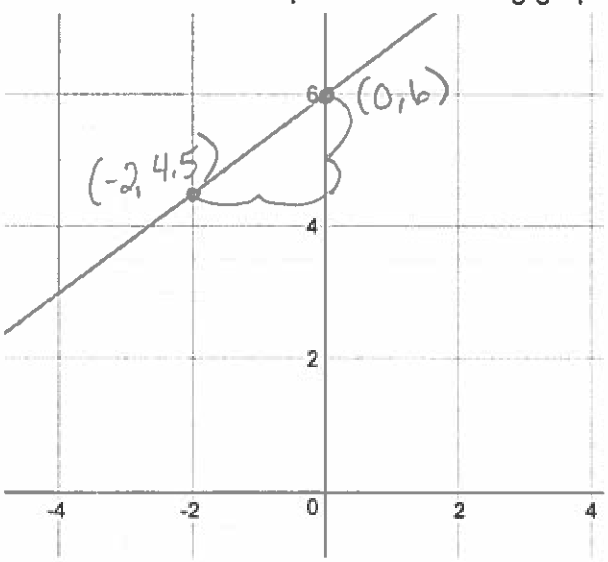
Chapter 7: Linear Equations and Graphs

Review (6.5 Slope)

1. How can you find the slope?

$m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

2. What is the slope of the following graph?



$m = \frac{rise}{run} = \boxed{\frac{1.5}{2}}$ or $\boxed{\frac{3}{4}}$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4.5}{0 - (-2)}$

$= \boxed{\frac{1.5}{2}}$ OR $\boxed{\frac{3}{4}}$

review for 7.1

3. Determine the slope of the following:

a. $y = 5x - 3$

Slope = 5

b. $2y = -6x + 3$ $\longrightarrow$ $\frac{2y}{2} = \frac{-6x}{2} + \frac{3}{2} \rightarrow y = -3x + 3/2$

Slope = 3

c. $y = 0.5x - 2.5$

Slope = 0.5

7.1 Slope-Intercept Form

Outcome: 3. Demonstrate an understanding of slope with relations to rise and run, and line segments and lines
6. Relate linear relations expressed in **slope-intercept form** to their graphs.
7. Determine the equation of a linear relation using a graph and a point and the slope to solve problems.

Definitions:
Y-Intercept: the y-coordinate of the point where a line or curve crosses the y-axis
You can find the value of y when x=0

Example: $y = 2x - 12$ when $x = 0$, $y = -12$; $(0, -12)$

Slope-Intercept Form: the equation of a line in the form $y = mx + b$
- **m** = the slope
- **b** is the y-intercept

Parameter: a variable that has a constant value in a particular equation

Example: $y = 10m + 100$. Determine the value of the parameter b.

Solution: "the parameter, b, represents the y-intercept, which is equal to 100"

Example 1:
What are the slope and y-intercepts of each equation?

a) $y = 25x + 3$

$m = 25$

y-intercept = 3

b) $y = 5x$

$m = 5$

y-intercept = 0

c) $y = 7$

$m = 0$

y-intercept = 7

d) $y = \frac{1}{2}x - 4\frac{1}{2}$

$m = \frac{1}{2}$

y-intercept = $-4\frac{1}{2}$

e) $y = \frac{3}{5}x + 2$

$m = \frac{3}{5}$

y-intercept = 2

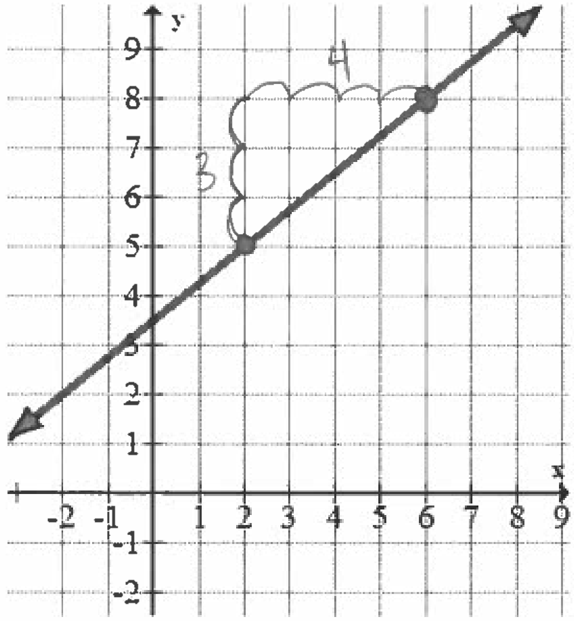
f) $y = 2x - 1$

$m = 2$

y-intercept = -1

Example 2:

a) What are the slope and y-intercept of the line shown in the graph?



$m = \frac{\text{rise}}{\text{run}}$

$= \frac{3}{4}$

y-intercept = 3.5

or

$\frac{7}{2}$

Steps
1) Find slope
2) Find y-intercept
3) write the equation

b) What is the equation of the line in slope-intercept form, y = mx + b?

$y = \frac{3}{4}x + \frac{7}{2}$

c) Use graph technology to check your equation.

→ move on to Extra Examples

Example 3:

Parents of members of the cheerleading squad rent a hall. They arrange a talent show as a fundraiser. The relationship between the number of tickets sold, x, and the profit, y, in dollars, may be represented by the equation 12x - y - 840 = 0.

a) What is the slope of the line? What does the slope represent?

m = 12 ; represents the cost per ticket

b) Identify the y-intercept. What does it represent?

b = -840 ; represents how much the ballroom costs.

c) How many tickets must the parents sell to reach the break-even point?

← zero profit

$y = 0$; $12x - 0 - 840 = 0$

$12x - 840 = 0$

$ +840 \quad +840$

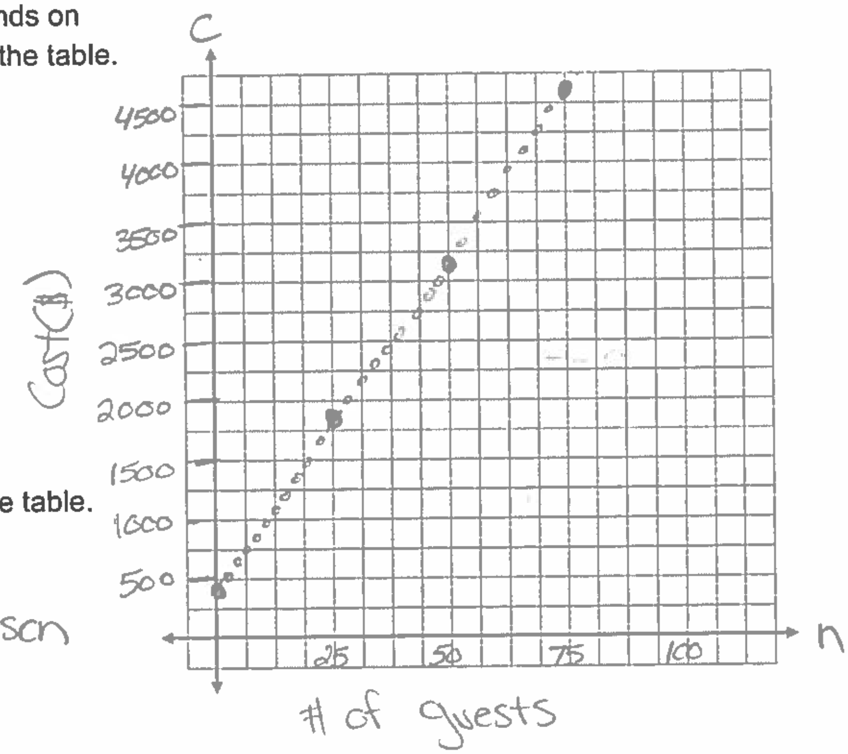
$\frac{12x}{12} = \frac{840}{12}$

x = 70 tickets

Example 4:

Asha has selected a hotel for her wedding reception. The cost involves a fee for the deluxe ballroom and a buffet charge that depends on the number of guests. This is shown in the table.

Number of Guests	Cost ($)
0	425
25	1800
50	3175
75	4550



a) Sketch a graph of the data in the table.

Discrete → can't have ½ a person

b) What are the slope and y-intercept of the line? What does each parameter represent?

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800-425}{25-0}$

$= \frac{1375}{25} = \boxed{55}$ → cost per person for the buffet

y-intercept $= \boxed{425}$ cost of the ballroom

① variables?
Cost → c
ppl → n

c) Write an equation that describes the relationship between the cost and the number of guests. Express the equation in slope-intercept form.

$C(n) = mn + b$

$C(n) = 55n + 425$

d) What is the cost for 140 guests?

$n = 140$ $\quad C = 55(140) + 425$

$\boxed{C = \$8125}$

e) Asha would like the total cost to be no more than $15,000. What is the maximum number of guests that can attend?

$C = 15,000$

$15,000 = 55n + 425$
$-425 \qquad -425$

$\frac{14575}{55} = \frac{55n}{55}$

$\boxed{n = 265 \text{ guests}}$

Example 5:

A decorator's fee can be modelled by the equation F = 75t + b. In the equation, F represents the fee, in dollars, t represents time, in hours, and b represents the cost of the initial consultation, in dollars.

a) Suppose the decorator spends 4 hours working for a client and charges the client $450. Determine the value of the parameter b.

$t = 4 \quad F = 450$

$450 = 75(4) + b$

$450 = 300 + b$

$-300 \quad -300$

$150 = b$

b) How many hours does the decorator work if a client is charged $975?

$F = 975$

$975 = 75t + 150$

$-150 \quad -150$

$825 = 75t$

$\frac{825}{75} = \frac{75t}{75}$

$\boxed{t = 11 \text{ hours}}$

Key Ideas

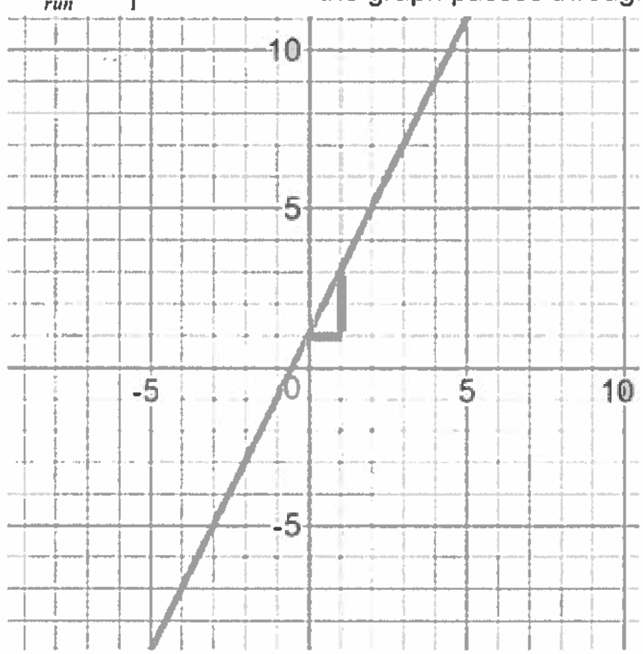
- The slope-intercept form of a linear equation is y = mx + b, where m is the slope of the line and b represents the y-intercept

  Example: y = 2x + 1

The slope = 2 $\quad \frac{rise}{run} = \frac{2}{1}$

y-intercept = 1

the graph passes through (0, 1)



Textbook Questions: pg. 349 # 1 - 13, 15, 17

7.2 General Form

Outcome: 6. Relate linear relations expressed in **general form** to their graphs.

Definitions:

General Form: the equation of a line in the form Ax + By + C = 0

- Where A, B, and C are real numbers
- A and B are **not both** zero.
- By convention, A is a whole number. This means that A will **always** be positive

Example: 3x + 5y - 11 = 0 → how do we know this is still an equation? "has exponent of 1"

X-intercept: the x-coordinate of the point where a line or curve crosses the x-axis.

- The value of x when y = 0

Example: y = 2x - 12,   when y = 0, x = 6 ; (6, 0)

Converting Slope-Intercept to General Form

① Clear fractions
→ multiply everything by the least common multiple in the denominator
② Move everything to one side

Example 1:

Rewrite the equation in general form, Ax + By + C = 0

a) $4(y) = \left(\frac{3}{4}x - 2\right)4$

$4y = 3x - 8$
$-4y \qquad -4y$
$\boxed{0 = 3x - 4y - 8}$

b) $y = -x - 5$

$+x+5 \qquad +x+5$

$\boxed{x + y + 5 = 0}$

c) $40(y) = \left(\frac{2}{5}x + \frac{1}{8}\right)40$

$40y = 16x + 5$

$\boxed{0 = 16x - 40y + 5}$

d) $y = \frac{4}{3}x + 6$

$(3)y = \left(\frac{4}{3}x + 6\right)3$

$3y = 4x + 18$
$-3y \qquad -3y$
$0 = 4x - 3y + 18$

Example 2:

Consider the linear equation 4x + 5y - 20 = 0

a) What is the x-intercept of a graph of the equation?

when $y = 0$

$4x + 5(0) - 20 = 0$
$+20 \qquad +20$

$\frac{4x}{4} = \frac{20}{4}$ $\boxed{x = 5}$

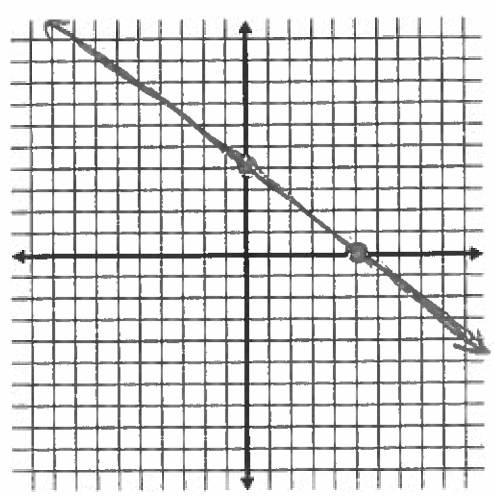
b) What is the y-intercept of the graph of the equation?

when $x = 0$

$4(0) + 5y - 20 = 0$
$+20 \qquad +20$

$\frac{5y}{5} = \frac{20}{5}$ $\boxed{y = 4}$

c) Use the intercepts to graph the line.



d) what is the slope?

$\frac{5y}{5} = \frac{-4x}{5} + \frac{20}{5}$

$\boxed{y = -\frac{4}{5}x + 4}$

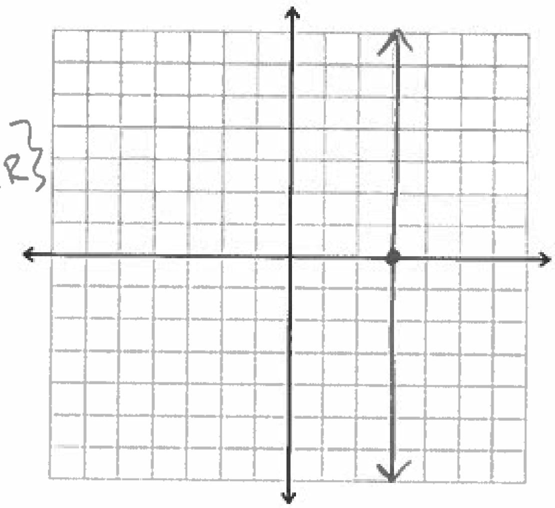
Example 3:

Sketch each linear relation and identify the intercepts. What are the domain and range for each relation?

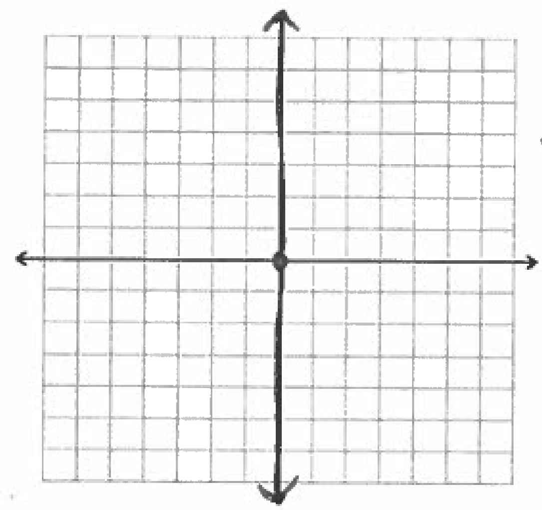
a) $x - 3 = 0$ $\quad x = 3$

D: $x = 3$

R $= \{y \mid y \in R\}$



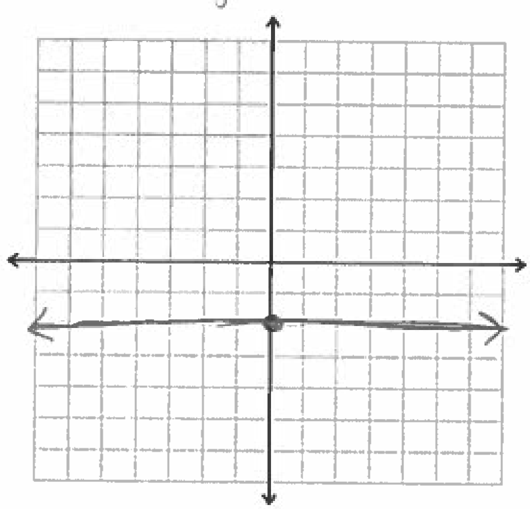
b) $x = 0$



D: $x = 0$

R $= \{y \mid y \in R\}$

c) $y + 2 = 0$ $\quad y = -2$



D $= \{x \mid x \in R\}$

R $=$ $y = -2$

Example 4:

Attempt
check
own first

Brooke wants to save \$336 to decorate her bedroom. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns \$12 per hour. On weeknights, she earns \$16 per hour working as a high school tutor.

a) Write an equation to represent the number of hours Brooke needs to work as a snowboard instructor, S, and as a tutor, T.

$$12S + 16T - 336 = 0$$

b) What is the S-intercept of a graph of the equation? What does the S-intercept represent?

$T=0$

$12S+16(0)-336=0$
$\qquad\qquad +336 \quad +336$

$\frac{12S}{12}=\frac{336}{12}$

$S=28$ hrs → how much she has to work if she doesn't work any hours as a tutor.

c) What would the T-intercept be? What does it represent?

$S=0$

$12(0)+16T-336=0$
$\qquad\qquad +336 \quad +336$

$\frac{16T}{16}=\frac{336}{16}$

$T=21$ hrs → how much she has to work if she doesn't work any hours as a snowboard instructor

d) Suppose Brooke works 8 hrs as a snowboard instructor. How many hours will she need to work as a tutor?

$S=8$

$12(8)+16T-336=0$

$96+16T-336=0$

$16T+96-336=0$

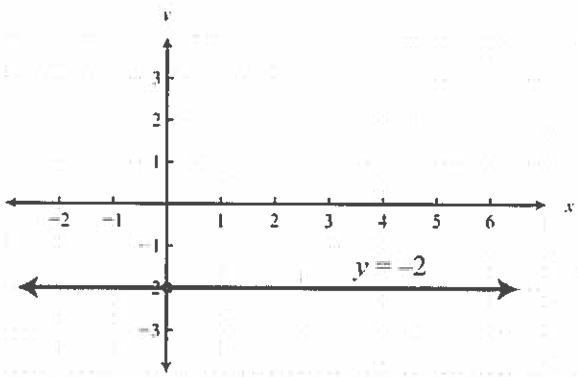
$16T-240=0$
$\qquad +240 \quad +240$

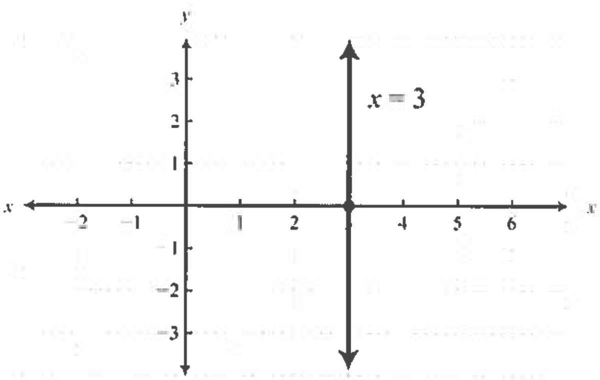
$\frac{16T}{16}=\frac{240}{16}$

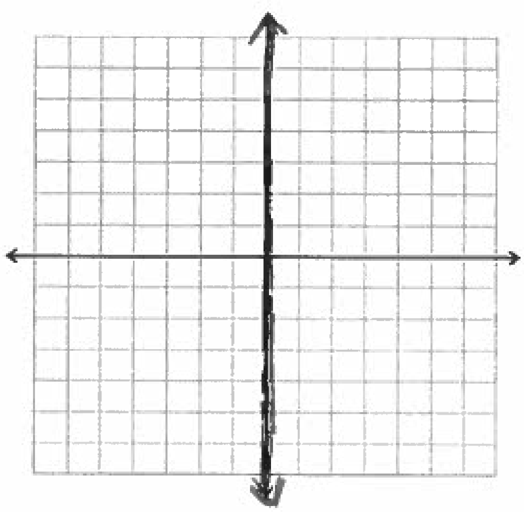
$\boxed{T=15 \text{ hrs}}$

Key Ideas

- The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, the draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x-intercepts, substitute $y = 0$ and solve.
  To determine the y-intercepts, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.







$x = 0$ → will be directly on the y-axis

Similarily, $y=0$ will be directly on the x-axis.

Textbook Questions: Pg. 365 # 1 - 8, 10, 13 -15

7.3 Slope-Point Form

Outcome: 6. Relate linear relations expressed in **slope-point form** to their graphs.

7. Determine the equation of a linear relation using a graph, a point and the slope, and two points to solve problems

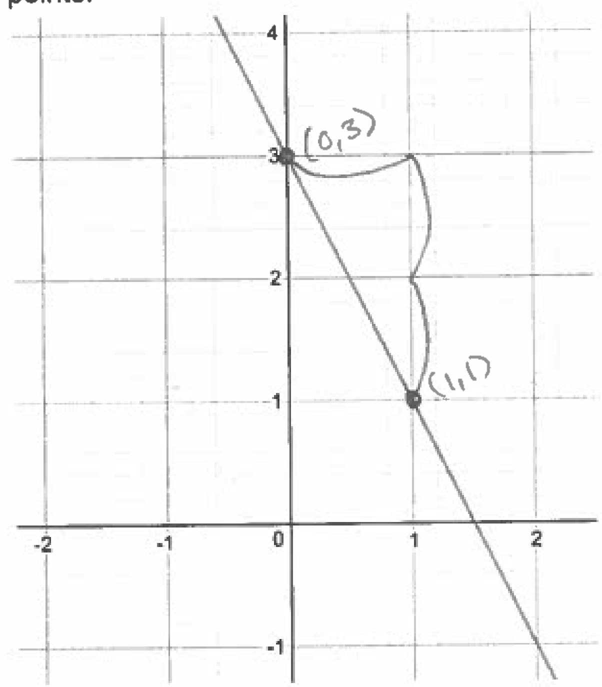
Definitions:

Slope-Point Form: the equation of a non-vertical line in the form $y - y_1 = m(x - x_1)$

- Where *m* is the slope, and (x_1, y_1) are the coordinates of a point on the line

Example 1:

Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of the line passing through the given points.



① $m = \frac{rise}{run} = \frac{-2}{1} = -2$

② Pick any point on the graph

$y - 1 = -2(x-1)$

OR

$y - 3 = -2(x - 0)$

$y - 3 = -2(x)$

b) convert the equation to general form Ax + By + C = 0, and slope-intercept form y = mx+b

General

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$
$\quad -y \; +1 \qquad -y + 1$

$0 = 2x - y - 1$

Slope-intercept

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$
$\quad +1 \qquad\quad +1$

$y = 2x - 1$

Example 2: **Writing an Equation of a Line Using Two Points**
Use slope-point form to write an equation of the line through (-5, 2) and (-2, 1). Then, write the equation in general form, Ax + By + C = 0

$y - y_1 = m(x - x_1)$

① $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-2-(-5)}$

$= \boxed{-\frac{1}{3}}$

② $y - 2 = -\frac{1}{3}(x - (-5))$

$\boxed{y - 2 = -\frac{1}{3}(x+5)}$

Convert

$y - 2 = -\frac{1}{3}(x+5)$

$y - 2 = -\frac{1}{3}x - \frac{5}{3}$

$-y \quad +2 \qquad -y \qquad +2$

$0 = -\frac{1}{3}x - y - \frac{5}{3} + 2\left(\frac{3}{3}\right)$

$0 = -\frac{1}{3}x - y - \frac{5}{3} + \frac{6}{3}$

$0 = {}^{-1}\left(-\frac{1}{3}x - y + \frac{1}{3}\right)$

$\boxed{0 = \frac{1}{3}x + y - \frac{1}{3}}$

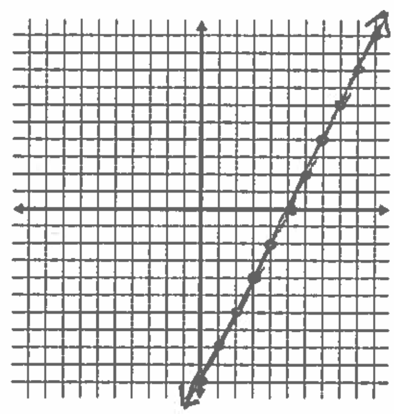
☆ m always is positive

Example 3: **Writing an Equation of a Line Using a Point and Slope**

a) Use slope-point form to write an equation of the line through (3, -4) with slope 2. Sketch a graph of the line.

$y - (-4) = 2(x - 3)$

$y + 4 = 2(x - 3)$



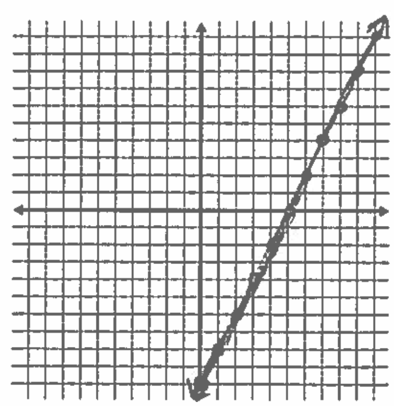
b) Express the equation in slope-intercept form, y = mx + b. Sketch a graph of this line.

$y + 4 = 2(x - 3)$

$y + 4 = 2x - 6$

$\quad -4 \qquad\quad -4$

$\boxed{y = 2x - 10}$



c) Compare your graphs.

They are the same.

Example 4:

A family drives at a constant speed from Calgary, AB, to visit relatives in Edmonton, AB. When they start driving at 9:00 am, they are 300km away from Edmonton. At 10:30am they reach Red Deer located 154 km from Edmonton.

a) Write an equation that describes the distance, *d*, in kilometers, from Edmonton in terms of *t* hours past 9:00 am.

$(0, 300) \quad (1.5, 154)$

$$m = \frac{\Delta d}{\Delta t} = \frac{154-300}{1.5-0} = -\frac{146}{1.5} = -\frac{292}{3}$$

$$d - d_1 = m(t - t_1)$$

$$d - 300 = -\frac{292}{3}(t - 0)$$

$$d - 300 = -\frac{292}{3}t$$

b) What time will the family reach Edmonton?

$d = 0 \text{ km}, \ t = ?$

$$0 - 300 = -\frac{292}{3}t$$

$$\frac{-300}{(-292/3)} = \frac{-\frac{292}{3}t}{(-292/3)}$$

$$\boxed{3.08 \text{hrs} = t}$$

Key Ideas

- The slope-point form of a non-vertical line in the form $y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) are the coordinates of a point on the line.
- An equation written in slope-point form can be converted to either slope-intercept form or general form
- Any point on a line can be used when determining the equation of a line in slope-point form.

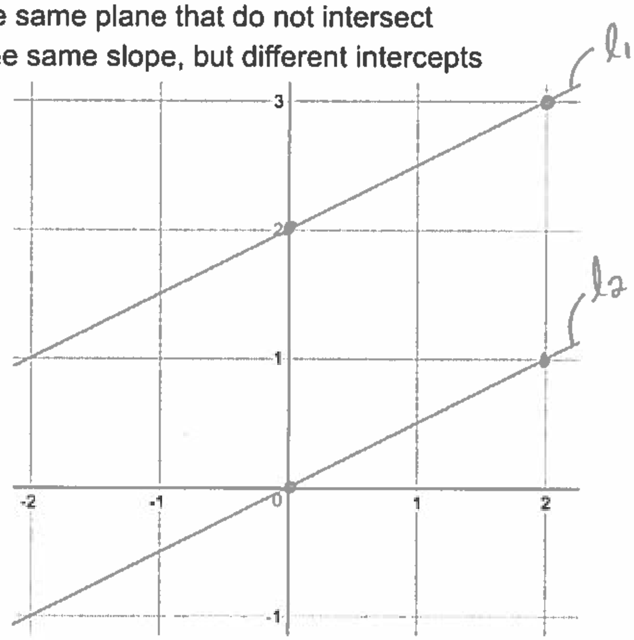
Textbook Questions: Pg. 377 #1 - 3, 5 - 8, 10 - 14

7.4 Parallel and Perpendicular Lines

Outcome: 3. Demonstrate an understanding of slope with relations to parallel lines and perpendicular lines.
7. Determine the equation of a linear relation using a point and the equation of a parallel or perpendicular line to solve problems.

Definitions:
Parallel Lines: lines in the same plane that do not intersect

- Lines that have the same slope, but different intercepts

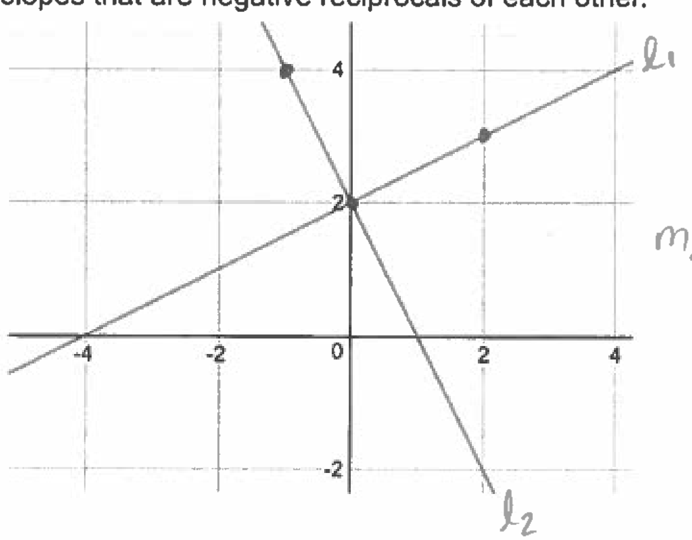


$m_1 = \frac{rise}{run} = \frac{1}{2}$

$m_2 = \frac{rise}{run} = \frac{1}{2}$

Perpendicular Lines: two lines that intersect each other at right angles (90 degrees)

- Lines that have slopes that are negative reciprocals of each other.



$m_1 = \frac{rise}{run} = \frac{1}{2}$

$m_2 = \frac{rise}{run} = \frac{-2}{1} = -2$

☆ DON'T HAVE TO HAVE THE SAME Y-INTERCEPT

Example 1:

State the slopes of lines that are parallel and lines that are perpendicular to each linear equation.

a) $y = 3x + 5$

Parallel

$y = 3x + 7$

Perpendicular

$y = -\frac{1}{3}x + 5$

b) $2x - 5y - 10 = 0$ → $\frac{2x}{5} - \frac{10}{5} = \frac{5y}{5}$

$\frac{2}{5}x - 2 = y$

Parallel

$\frac{2}{5}x + 2 = y$

Perpendicular

$-\frac{5}{2}x - 2 = y$

Example 2:

Determine whether the lines in each pair are parallel, or perpendicular, or neither.

a) $y = \frac{1}{2}x - 7$
$y = 2x - 7$

niether

b) $y = 3x - 4$
$y = 3x + \frac{1}{4}$

parallel

c) $y = \frac{2}{5}x - 6$
$5x + 2y = 8$

perpendicular

$\frac{2y}{2} = -\frac{5x}{2} + \frac{8}{2}$

$y = -\frac{5}{2}x + 4$

Example 3:

Write an equation in slope-intercept form of a line that is parallel to 3x + y + 3 = 0 and passes through (5, -6). Use technology to verify that the lines are parallel.

$y - y_1 = -3(x - x_1)$

$y - (-6) = -3(x - 5)$

$y + 6 = -3x + 15$
$\quad -6 \qquad\qquad -6$

$\boxed{y = -3x + 9}$

Example 4:

A line is perpendicular to 4x + y - 12 = 0 and passes through (8, -6). Write the equation of the line in both slope-intercept form and general form.

$y = -4x + 12$

$y - y_1 = \frac{1}{4}(x - x_1)$

$y - (-6) = \frac{1}{4}(x - 8)$

$y + 6 = \frac{1}{4}x - 2$

$-6 \qquad -6$

$\boxed{y = \frac{1}{4}x - 8}$

General

$y = \frac{1}{4}x - 8$

$-y \qquad -y$

$\boxed{0 = \frac{1}{4}x - y - 8}$

Key Ideas

- Parallel lines have the same slope and different intercepts. Vertical lines are parallel to each other, as are horizontal lines, if they have different intercepts.
- Perpendicular lines have slopes that are negative reciprocals of each other. A vertical line with an undefined slope and a horizontal line with a slope of zero are also perpendicular.
- The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.
    - A line perpendicular to $y = 5x + 7$ has the same y-intercept. The line $y = 5x + 7$ has a slope of 5 and a y-intercept of 7. The perpendicular line has a slope of -⅕ and a y-intercept of 7. So, the equation of the perpendicular line is $y = -\frac{1}{5}x + 7$

Textbook Questions: Pg 390 # 1-8, 10-13, 15