Name: Key

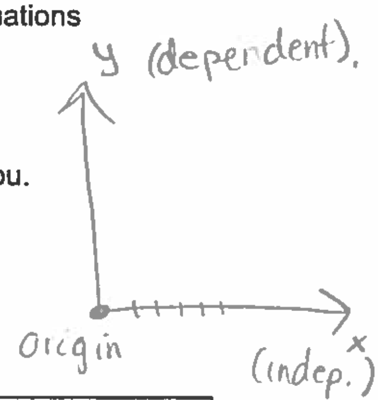
Chapter 6: Linear Relations and Functions

6.1 Graphs of Relations

Outcomes: 1. Interpret and explain the relationships among data, graphs and situations.
4. Describe and represent linear relations using:

- Words
- Ordered pairs
- Tables of values
- Graphs
- equations

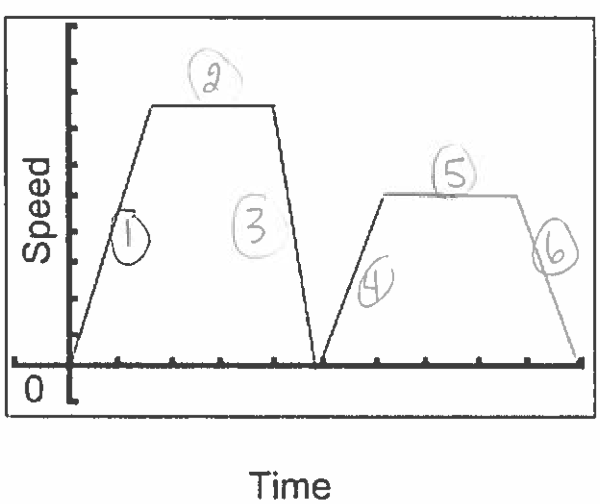
Investigation: Work through pg. 268 #1-4.
QUESTION 3: Stay in your pairs, then switch with another pair around you.



Linking the Ideas:

- Graph is a great way to show the relationship between two quantities.
- Constant rate of change is demonstrated through a line, it may be increasing or decreasing
- Not all relationships are represented by a straight line, a curve represents the rate of change isn't constant
- A horizontal line means there is no rate of change

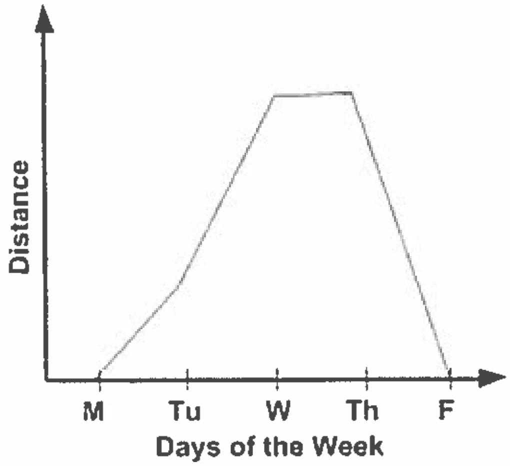
Example 1: Interpret a Graph
The graph shows the speed of the boat that is pulling a wakeboarder. Describe what the boat is doing.



1) The boat accelerates (starts moving)
2) The boat maintains speed for a duration of time
3) Boat slows down to 0.
4) Boat accelerates to a slower speed
5) Mainstains speed
6) Slows down to 0

Example 2:

The graph shows Janes 5 day road trip. Describe what she was doing each day.



M : travelling some distance

Tu : longer travel distance to her destination

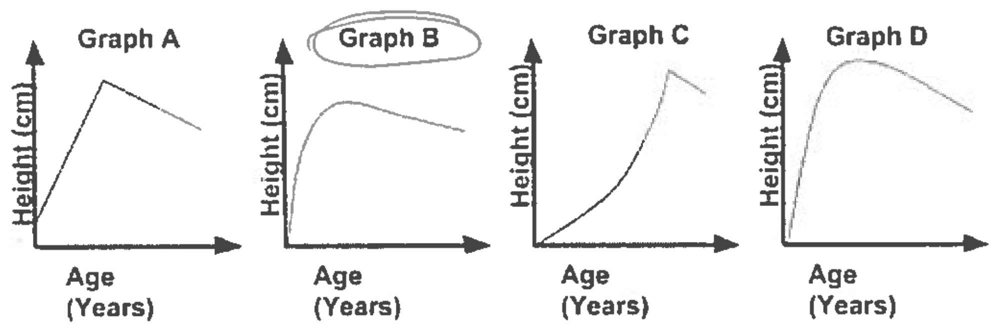
W : Stayed where she was

Th : Travelled home

F : Arrived home.

Example 3:

Which graph best represents a person's height as the person ages? Explain your choice.

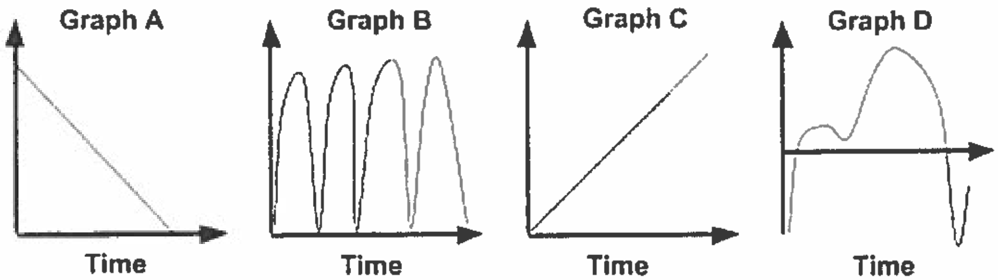


height increases as one gets older. Then, when one reaches physical maturity, height remains the same until the senior years when height decreases slightly.

Example 4:

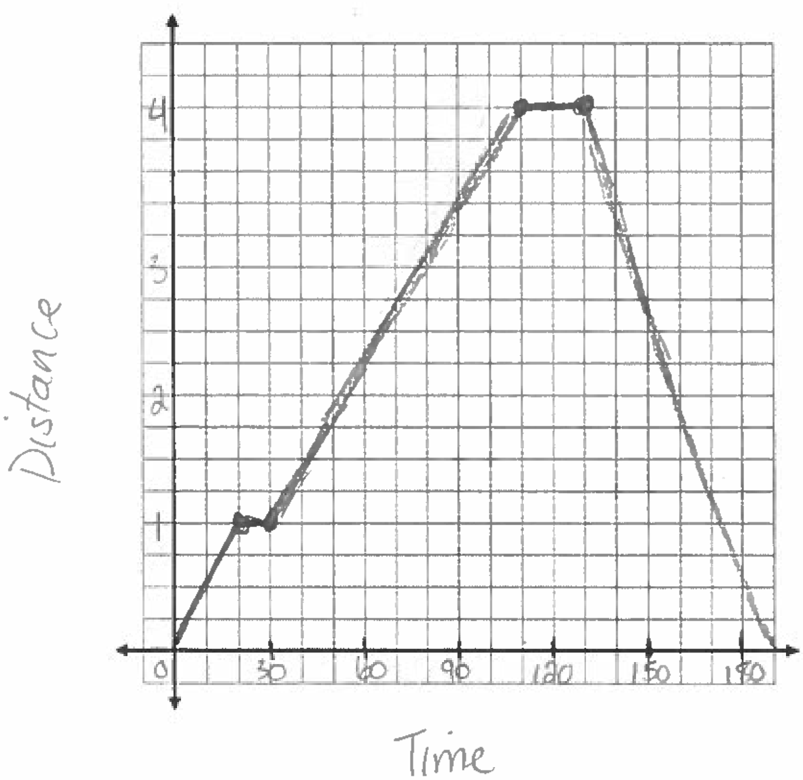
Match each scenario with its proper graph.

1) The temperature of Lethbridge's yearly weather. D
2) The speed of a car decelerating. A
3) The distance of Shay walking to her friends house. C
4) A basketball's height as it travels across the gym. B



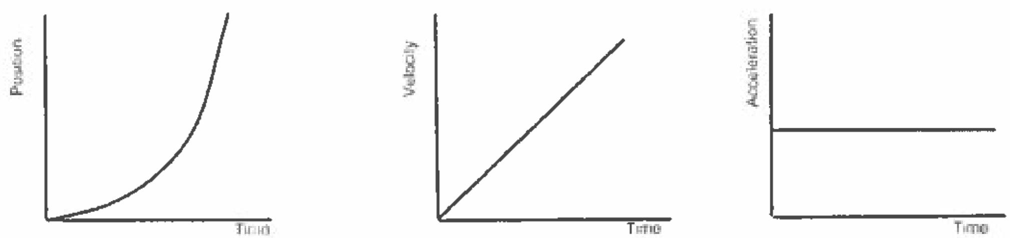
Example 5:

Garrett starts his day by going for a morning run. He runs to a convenience store that is 1km away in 20 minutes. He stays in the convenience store for 10 minutes and buys a water bottle. Then he jogs to a dog park which is 3km from the store, which takes him 80 minutes. He runs into his friends and stays for 20 minutes to catch up. Then in 60 minutes he runs the same path home. Using the information above, draw a distance-time graph that shows Garrett's distance from his house. Explain each section of your graph.



Key Ideas

- When comparing two quantities, straight lines are used to indicate a constant change in the relationship
- Curves are used to demonstrate there is no constant rate of change
- Horizontal lines are used if one quantity is not changing relative to a change in the other quantity.



Motion Map

Textbook Questions: Pg. 274 # 1 - 3, 6 - 9

6.2 Linear Relations

Outcomes: 2. Demonstrate an understanding of relations and functions.
8. Represent a linear function, using function notation.

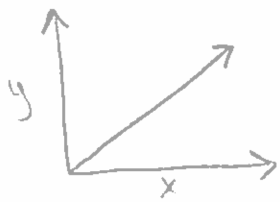
Definitions:

Relation: an association between two quantities.
Can be expressed in words, equations, ordered pairs, table of values or a graph.

ie.) 3 times the distance, d, is equal to the time, t. "$t = 3d$" equation

words

Linear Relation:

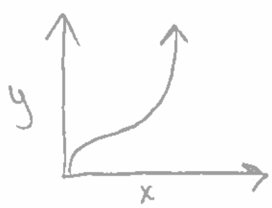


A relation that forms a straight line when the data is plotted on a graph.

ex.) $x = 7$ $3m + 4n = 7$

Non-ex.) $x^2 + 4 = y$ $2m^2 + 4n^2 = 1$ $xy = 4$

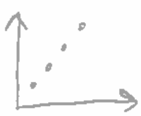
Non-Linear Relation:



A relation that doesn't form a straight line when the data is plotted on a graph.

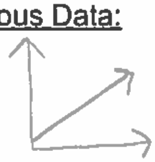
ex.) $x^2 + 4 = y$; $2m^2 + 4n^2 = 1$; $xy = 4$

Discrete Data:



Data values on a graph that ARE NOT connected
defined as a set of ordered pairs

Continuous Data:



Data values on a graph that ARE connected

Independent Variable:

Variable for which values are selected; variable doesn't change when another variable is added.

Dependent Variable:

Variable whose values depend on those of the independent variable.

independent "input"

x	$y = 2x$
1	2
2	3
3	6

dependent "output"

HOW TO GRAPH ON A CALC

↳ "Y =" add equation

↳ "Graph" to go to graph

can't see your graph? check "window"

Xmin = -10
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscle = 1

Example 1:

Another popular event at *Les Folies Grenouilles* is the fireworks display. Assume that the event organizers send off 20 firework shells each minute.

a) Is the relationship between the total number of fireworks and the duration of the event linear or non-linear? Explain.

linear because 20 fireworks are being sent off every minute. Thus, the number of fireworks that have been seen off increases by 20 each time.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

t = Time = independent

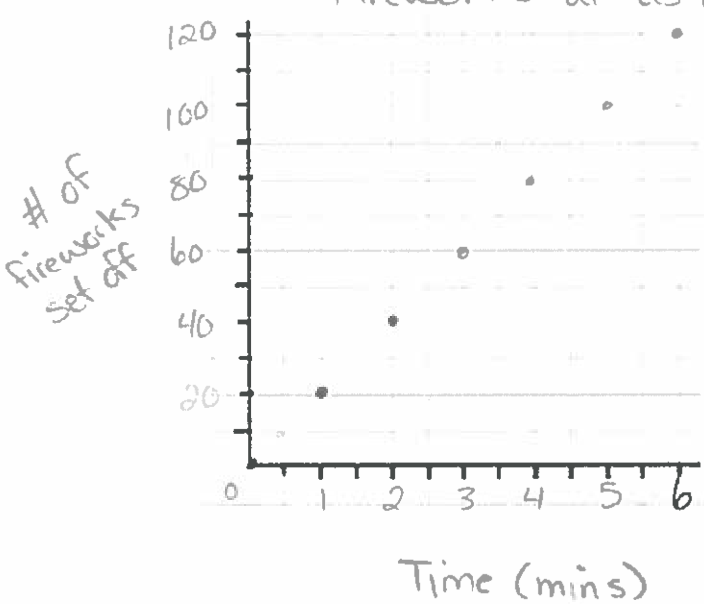
f = number of fireworks = dependent

c) Create a table of values for this relation. What are appropriate values for the independent variable?

t	f
0	0
1	20
2	40
3	60
4	80

d) Create a graph for the relation. Are the data discrete or continuous?

Fireworks at Les Folies



The data is discrete, you can't have half a firework

ordered pairs:

{(0,0), (1,20), (2,40), (3,60), (4,80), (5,100), (6,120)}

Example 2:

Determine whether each relation is linear. Explain why or why not.

a) The relationship between the cost to rent a dance hall and the number of people attending the dance, if the hall charges \$200 plus \$5 for each person who attends.

Yes since the overall cost of rent depends on the number of people attending, even cost per person.
- when a new person attends, \$5 is added to cost.

b) The relation described by the equation $x^2 + y^2 = \$25$

The degree is 2 so it's not linear

Graph & the equation is a circle.

c) The relation described by the set of ordered pairs {(10, 12,), (15, 4), (20, -4), (25, -12), (30, -20)}

The relation is linear, the independent variable increases by 5 every time, and the dependent variable decreases by 8.

Example 3:

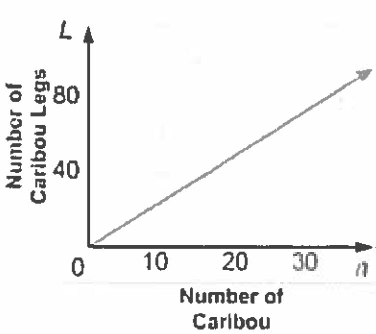
There is a linear relationship between the number of caribou, *n*, in a herd and the number of caribou legs, *L*. Which representation models this relation?

A $L = 4n$

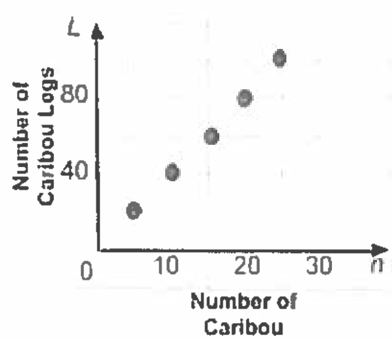
B (0, 0), (3, 12), (8, 32), (15, 60), (50, 200)

C $L = n + 4$

D



E



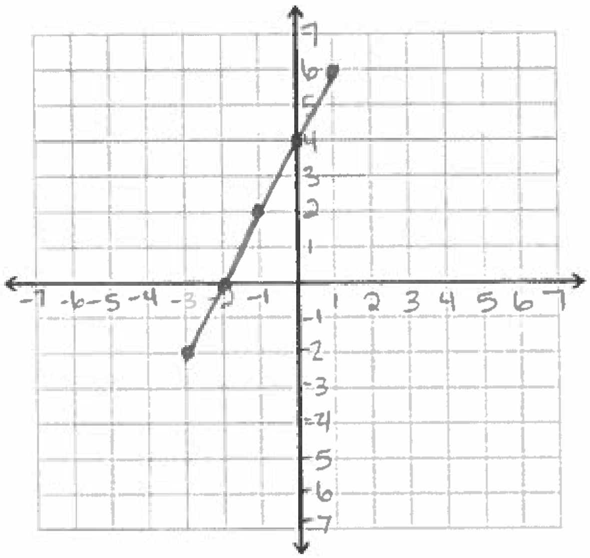
F

n	*L*
3	6
6	12
9	18
12	24

Example 4:

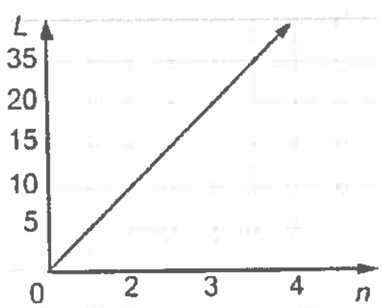
Convert each relation from its current representation to the one suggested. Then state whether it is linear or non-linear.

a) (-3, -2), (-2, 0), (-1, 2), (0, 4), (1, 6) to a graph



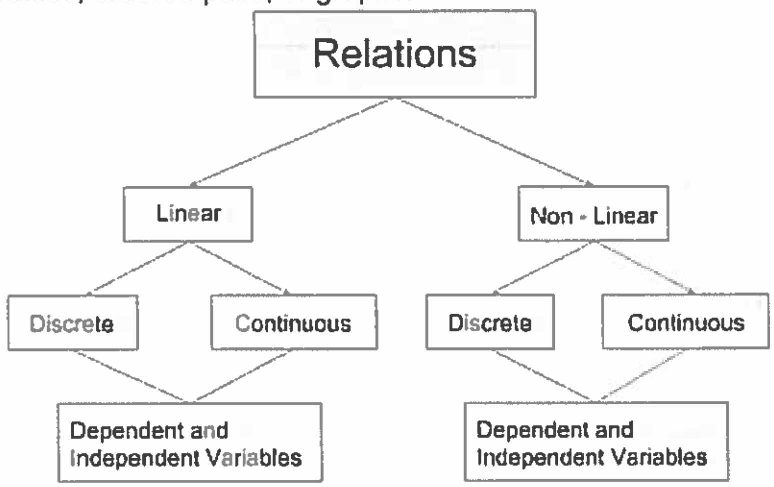
linear

b) To a table of values



x	y
0	0
1	5
2	10
3	15
4	20
⋮	⋮

linear

Key Ideas
• Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs. 

Textbook Questions: Pg. 287 # 1-5, 7

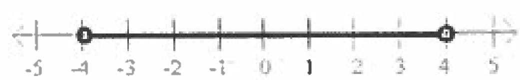
6.3 Domain and Range

Outcome: 1. Interpret and explain the relationships among data, graphs and situations

Definitions:

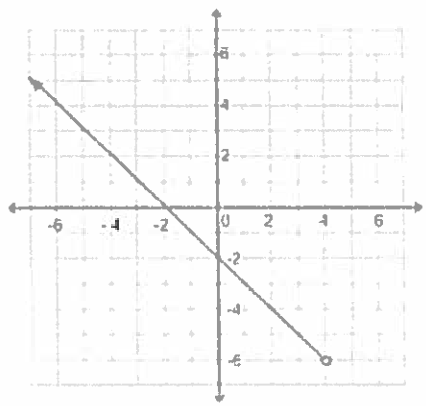
Domain: set of all possible values for the independent variable in a relation (x)

Range: set of all possible values for the dependent variable in a relation (y)

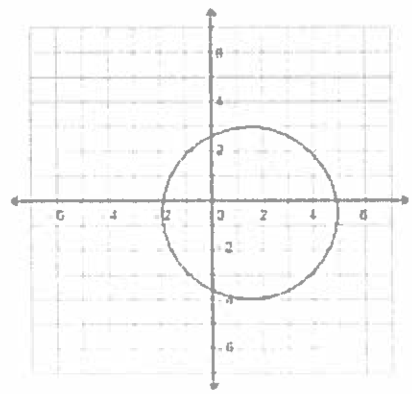
Multiple Ways of Writing Domain and Range	
Ways of Expressing	**Example**
Words: describe the values that are allowed.	The domain is the set of all real numbers between 0 and 12, inclusive. The range is the set of all real numbers greater than 20.
Number Lines: a picture of the values that are allowed. Closed points = these points are included in the domain and range Open points = points are not included in the domain and range	
List: gives the domain and range for discrete data when there are not many numbers in the set.	For the relation (0,0), (1, 3), (2, 3), (3, 5), the domain is {0, 1, 2, 3}, and the range is {0, 3, 5}
Set Notation: formal mathematical way to give the values of the domain and range.	The domain: {x | x ≤ 10, x E R} { } is the type of brackets to use for a set. E means "is an element of" | means "such that" Therefore, the statement read: x is an element of real numbers such that x is less than or equal to 10.
Interval Notation: uses brackets to indicate the interval. Rules: - The bracket "] " is used if the end number is INCLUDED - The bracket ") " is used if the end number is NOT included - The infinity symbol, ∞ , is used if there is no endpoint (go ones forever).	A domain of all numbers between -2 and 5, inclusively, would be [-2, 5]. A range of all numbers greater than 10 would be (10, ∞)

Example 1:
For each graph, give the domain and range using words, a number line, interval notation, and set notation.

a)



b)



a)

Words

- Domain: the set of all real numbers between $-\infty$ and 4, but doesn't include 4.
- Range: the set of all real numbers between -6 and ∞, but doesn't include -6.

Number line

- Domain

- Range

Interval Notation

- Domain: $[-\infty, 4)$
- Range: $(-6, \infty]$

Set Notation

- Domain $\{x \mid x < 4, x \in R\}$
- Range $\{y \mid y > -6, y \in R\}$

b)

Words

- Domain: the set of all real numbers between -2 and 5.
- Range: the set of all real numbers between -4 and 3.

Number Line

- Domain

- Range

Interval Notation

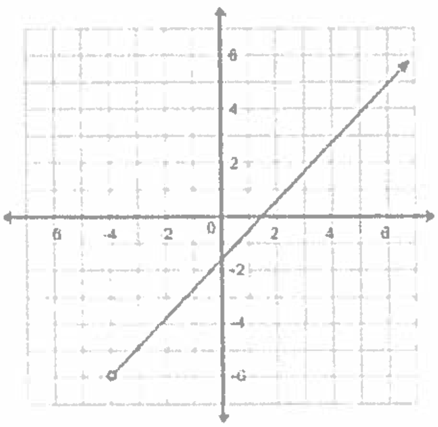
- Domain: $[-2, 5]$
- Range: $[-4, 3]$

Set Notation

- Domain $\{x \mid -2 \leq x \leq 5, x \in R\}$
- Range $\{y \mid -4 \leq y \leq 2, y \in R\}$

Example 2:

For each graph, give the domain and range using words, a number line, interval notation, and set notation.



a)

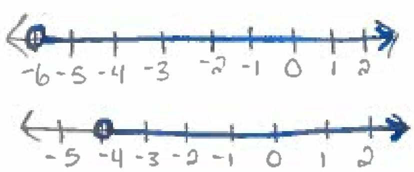
Words

- Domain: the set of all real numbers from −4 to ∞, but doesn't include −4
- Range: the set of all real numbers from −6 to ∞, but doesn't include −6.

Number line:

R =

D =



Interval:

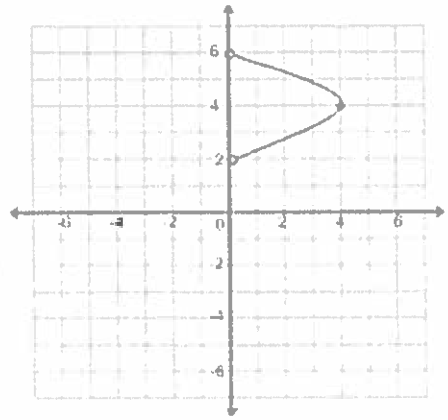
D = $(-4, \infty]$

R = $(-6, \infty]$

Set Notation:

D = $\{x \mid -4 < x, x \in R\}$

R = $\{y \mid -6 < y, y \in R\}$



b)

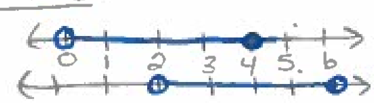
Words

- Domain: the set of all real numbers from 0 to 4, but not including 0.
- Range: the set of all real numbers from 2 to 6, but not including 2 or 6.

Number line:

D =

R =



Interval:

D = $(0, 4]$ R = $(2, 6)$

Set Notation

D = $\{x \mid 0 < x \le 4, x \in R\}$ R = $\{y \mid 2 < y < 6, y$

Example 3:

Data for a relation are recorded in the table of values. Give the domain and range using set notation and lists.

x	y
-3	5
-2	6
-1	7
0	8
1	9
2	10

D = $\{x \mid -3 \le x \le 2, x \in R\}$

R = $\{y \mid 5 \le y \le 10, y \in R\}$

$(-3,5), (-2,6), (-1,7), (0,8), (1,9), (2,10$

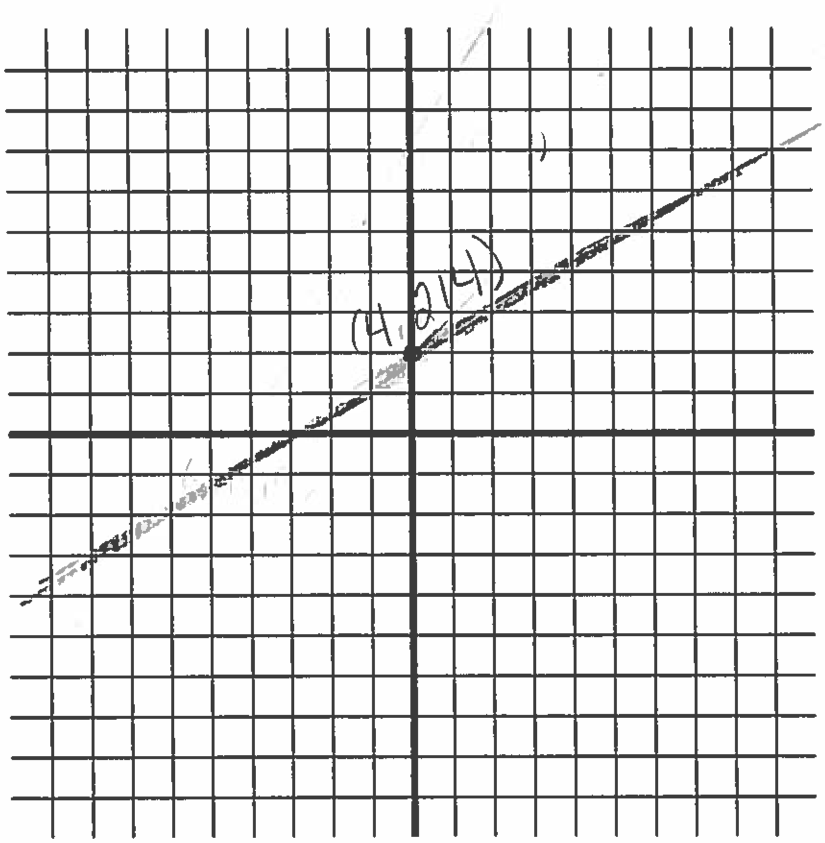
Domain = $\{-3, -2, -1, 0, 1, 2\}$

Range = $\{5, 6, 7, 8, 9, 10\}$

Window
Xmin = 42
Xmax = 63
Ymin = 424
Ymax = 529

Example 4:

The same species of corn grows at an average rate of 5 cm per day from the start of week 7 until the end of week 9. The plant's growth in this period is modelled using the formula $h = 5a + 214$, where h is the height of the plan, in centimeters, and a is the number of days since the start of week 7. Using a graphing calculator to show a graph of the plant's height for these three weeks.



Give a rough drawing.

$\{x \mid 0 \leq x, x \in R\}$

$\{y \mid 214 \leq y, y \in R\}$

Key Ideas

- Domain of a relation is the set of all real numbers for which the independent variable (first coordinates, first column, x-axis) is defined.
- Range of a relation is the set of all real numbers for which the dependent variable (second coordinates, second column, y-axis) is defined.
- There are different ways of expressing domain and range:
  - Words
  - Number Line
  - Interval Notation
  - Set Notation
  - A List

Textbook Questions: Pg. 301 # 1 - 2(a,c,e), 3 - 9.

6.4 Functions

Outcome: Demonstrate an understanding of slope with respect to:

- Rise and run
- Line segments and lines
- Rate of change
- Parallel lines
- Perpendicular lines

Definitions:

Function: relation in which each value of the independent variable is associated with exactly one value of the dependent variable

ie.) $y = x + 5$

Function Notation:

a symbolic notation used for writing a function

- $f(x)$ reads as "f of x" or "f at x"   ie.) $f(x) = x + 5$

Vertical Line Test:

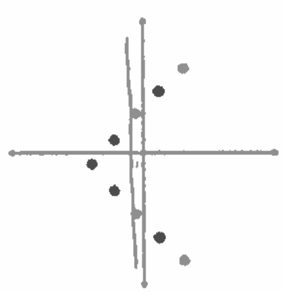
- a test to see if a graph represents a function

- if any vertical line intersects at more than one point on the graph, the relation is <u>NOT</u> a function.

Example 1:

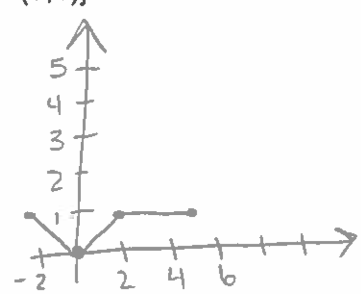
Which of the following relations are functions? Explain your choices.

a)



$\boxed{\text{Not a function}}$

b) {(-2, 1), (0, 0), (2, 1), (5,1)}



$\boxed{\text{A function}}$

→ look for an x value thats given more than once

$\{((-2)\,1)\,(-2)\,2),\ (0,0)$

c)

x	y
1	3
2	3
3	4
4	4
5	4

$\boxed{\text{A Function}}$

Example 2:

The function F(C) = 1.8C + 32 is used to convert a temperature in degrees Celsius (°C) to a temperature in degrees Fahrenheit (°F).

a) Determine F(86). Explain your answer.

$C = 86$ $\quad F(86) = 1.8(86) + 32$

$\boxed{F(86) = 186.8°\ F}$

b) Determine C so that F(C) = 98.6. Explain your answer.

$F(C) = 98.6$

$98.6 = 1.8C + \cancel{32}$

$-32 \qquad \cancel{-32}$

$\frac{66.6}{1.8} = \frac{\cancel{1.8}C}{\cancel{1.8}}$

$\boxed{C = 37°C}$

c) Another measurement scale for temperature that is used in science is the Kelvin scale. The function K(C) = C + 273.15 can be used to convert from degrees Celsius to Kelvins. Determine K(80) and explain your answer.

$C = 80$

$K(80) = 80 + 273.15$

$\boxed{K(80) = 353.15\ K}$

Thus, when the temperature is 80°C, it is 353.15 K.

Example 3:

If f(x) = -2x + 10, determine:

a) f(4) $\quad x = 4$

$f(4) = -2(4) + 10$

$F(4) = -8 + 10$

$\boxed{f(4) = 2}$

b) f(-10) $\quad x = -10$

$f(-10) = -2(-10) + 10$

$f(-10) = 20 + 10$

$\boxed{f(-10) = 30}$

c) f(x) = 12

$12 = -2x + \cancel{10}$

$-10 \qquad \cancel{-10}$

$\frac{2}{-2} = \frac{\cancel{-2}x}{\cancel{-2}}$

$\boxed{x = -1}$

Example 4:

Use the relation y = 3x - 1.

a) Write the relation in function notation using *f* for the name of the function.

$f(x) = 3x - 1$

b) Make a table of values. Graph the function.

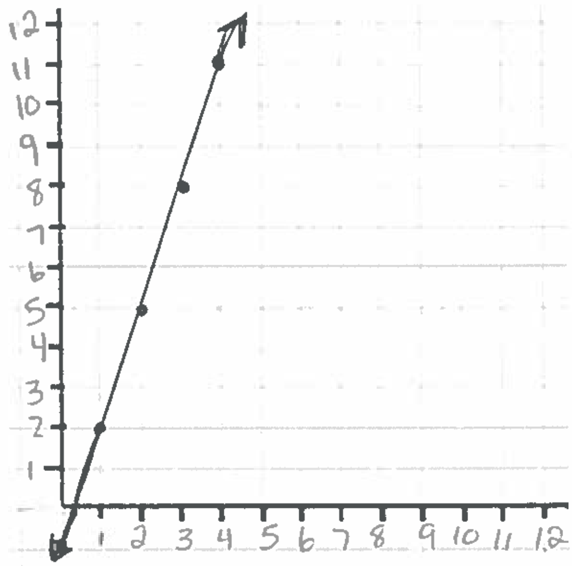
x	f(x)
0	-1
1	2
2	5
3	8
4	11
⋮	⋮

$3(0)-1 = -1$

$3(1)-1 = 2$

$3(2)-1 = 5$

$3(3)-1 = 8$



c) Determine the value of x if f(x) = 53.

$f(x) = 3x - 1$

$53 = 3x - \cancel{1}$

$+1 \qquad \cancel{+1}$

$\frac{54}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$\boxed{x = 18}$

Key Ideas
• All functions are relations but not all relations are functions. • Relation is classified as a function if each value in the domain corresponds to example one value in the range. • Each function has its own formula, or rule, that is often given using special notation, called **function notation**.

Textbook Questions: Pg. 311 #1-8, 10

6.5 Slope

Outcome: Demonstrate an understanding of slope with respect to:

- Rise and run
- Line segments and lines
- Rate of change
- Parallel lines
- Perpendicular lines

Definitions:

<u>Slope:</u> ratio of the vertical change (rise) to the horizontal change (run), of a line or line segment.

Slope = $\dfrac{\text{vertical change}}{\text{Horizontal change}}$ or $m = \dfrac{\text{rise}}{\text{run}}$ or $m = \dfrac{\Delta y}{\Delta x}$

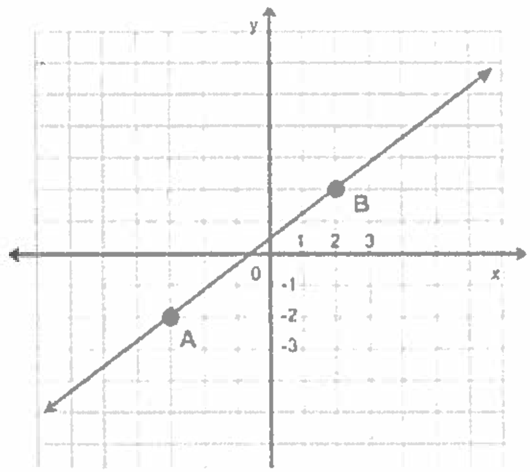
<u>How to find the slope on a graph</u>

Move from point A to point B:

$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{4}{5}$

Move from point B to point A:

$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{-4}{-5} = \dfrac{4}{5}$



<u>Conclusion:</u>

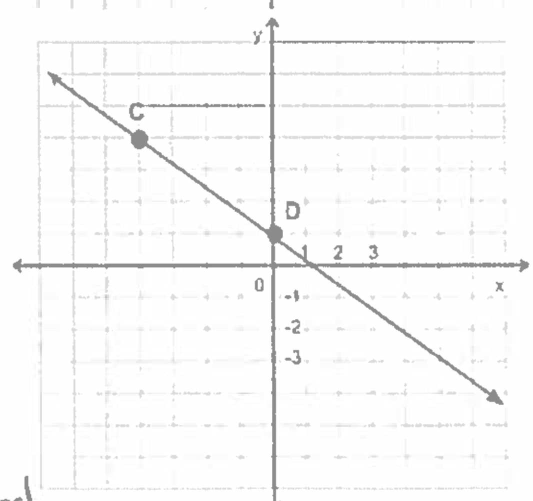
A line segment moving <u>upward</u> from left to right is <u>always positive</u>

Move from point C to point D:

$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{-3}{4} = -\dfrac{3}{4}$

Move from point D to point C:

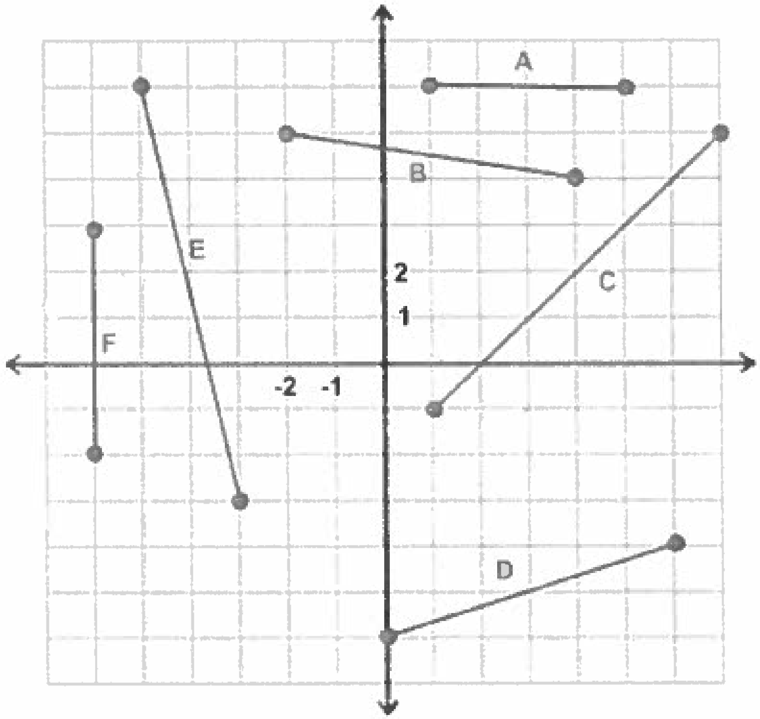
$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{-4}$



<u>Conclusion:</u>

A line segment moving <u>downward</u> from left to right is <u>always negative</u>

Example 1:
Classify the slope of each line segment is positive, negative, or neither.



A = neither

B = negative

C = Positive

D = Positive

E = Negative

F = Neither

Example 2:
Determine the slope of each line segment in the image above.

A = $\frac{0}{4} = 0$

B = $\frac{-1}{6}$

C = $\frac{6}{6} = 1$

D = $\frac{2}{6}$

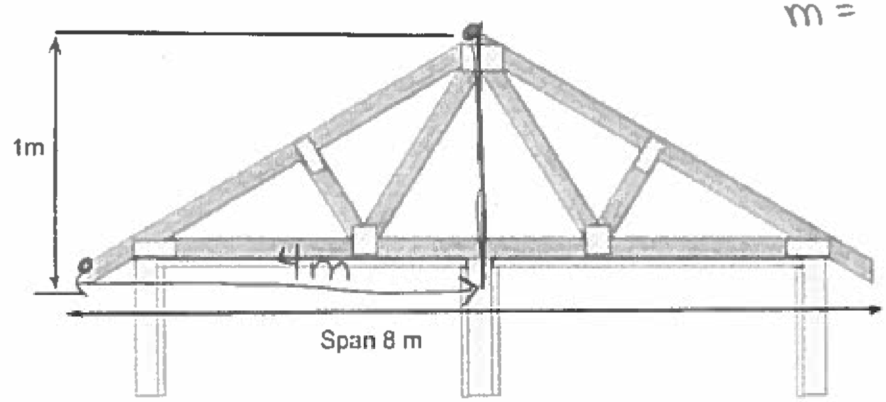
E = $-\frac{9}{2}$

F = $\frac{5}{0}$ = D.N.E

✱

Example 3:

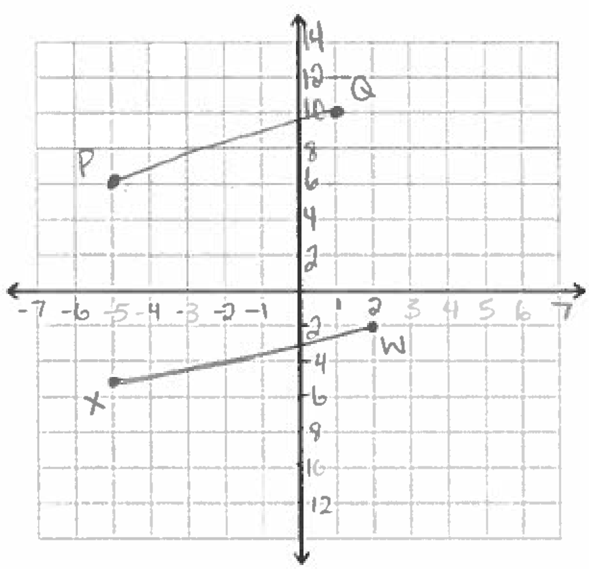
When discussing a roof truss, carpenters refer to the *span* instead of the *width*. They talk about the *pitch* rather than the *slope*. If a roof truss has a height of 1m and a span of 8m. Determine the pitch and explain your answer.



$$m = \frac{\text{rise}}{\text{run}} = \frac{1\text{ m}}{4\text{ m}} = \boxed{\frac{1}{4}}$$

Example 4:

a) Use a graph to determine the slope of a line segment with endpoint P(-5, 6) and Q(1, 10)



$$m_{PQ} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

b) Use the slope formula to determine the slope of the line segment with endpoints W(2, -2) and X(-5, 5).

$x_1 \quad y_1$

$x_2 \quad y_2$

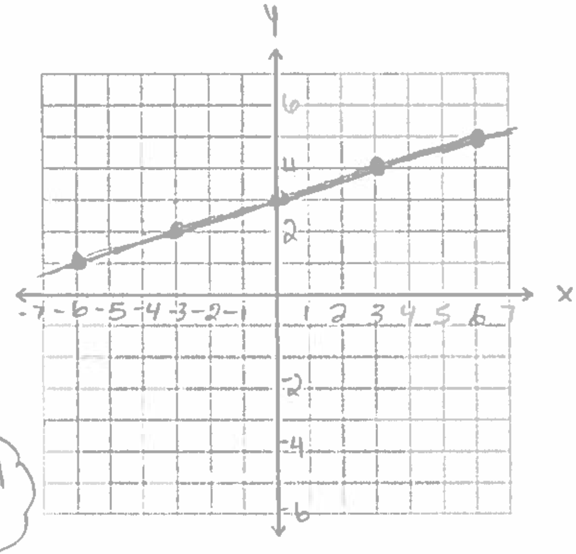
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-5-2} = \frac{7}{-7} = \boxed{-1}$$

Example 5:

The point (-6, 1) is on a line that has a slope of ⅓. List three other points on the line and graph the line.

$\frac{1}{3} = \frac{rise}{run}$ rise = 1

run = 3



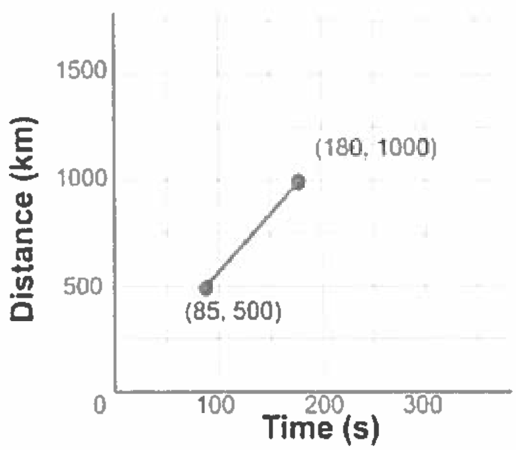
(-6, 1), (-3, 2), (0, 3),

(3,4), (6, 5)

add 3 to previous x, and
add 1 to previous y

Example 6:

The graph shows the approximate times at the 1000-m mark and at the 1500-m mark for a rowing crew of the girls' junior open eighth race at the Brentwood Regatta. Determine the average rate of change for this portion of the race.



$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

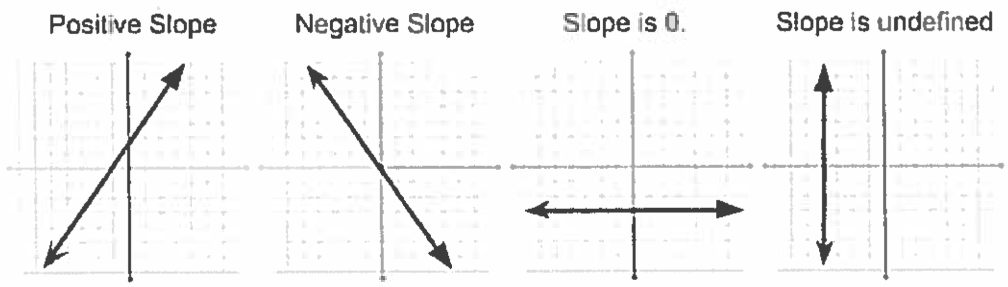
$$= \frac{1000 - 500}{180 - 85}$$

$$= \frac{500}{95} = 5.2631\ldots$$

$$\boxed{= \frac{100}{95}}$$

put into a fraction

Key Ideas



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- The slope of a line is the ratio of the rise over run.
- The slope of a line can be determined using two points on the line (x_1, y_1) and (x_2, y_2).
  - $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$
- If you know one point of the line, you can use the slope to find other points on the line.
- The slope gives the average rate of change.

Textbook Questions: Pg. 325 #1-8