Math 10C ~~2012-2013~~ Name: Key

Chapter 5: Polynomials

5.1 Multiplying Polynomials

Review:

1. Use the distributive property and simplify the following.

   a. $4(2y + 1)$

   $= 8y + 4$

   b. $3(x - 2) + 5(10)$

   $= 3x - 6 + 50$

   $= 3x + 44$

Outcome: Demonstrates an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials).

Definitions:

Polynomials: An expression that can have coefficients (like 4), variables (like x or y), and exponents (like 3 in y^3), that can be combined using addition, subtraction, multiplication and division, but

- The exponent of a variable can only be greater than 0 (i.e. x^{-2}) → Non-examples
- Can't be divided by a by a variable (i.e. $\frac{3}{x+3}$) →
- Can't have an infinite number of terms.

Binomial: A polynomial with two terms.

For example:

- $x + 3$
- $x - 3y$

Distributive Property: the rule that states $a(b + c) = ab + ac$

For example:

- $2(x + 5) = 2x + 10$

Trinomial: a polynomial with three terms.

For example:

- $x^2 + 2x - 5$
- $3x^2 - xy + y^2$

FOIL → (First/Outside/Inside/Last)
→ Only works with 2 binomials

Example 1:

Multiply the following binomials.

a) $(x - 3)(x - 5)$ $= x^2 - 5x - 3x + 15$

$= x^2 - 8x + 15$

$x \cdot -5$

b) $(5m - 1)(2m + 6)$ $= 10m^2 + 30m - 2m - 6$

$= 10m^2 + 28m - 6$

$1x^1(3x^2) = 3x^3$

Example 2:

Multiply the following binomials with a trinomial.

a) $(x - 4)(3x^2 + 8x - 6)$ $= 3x^3 + 8x^2 - 6x - 12x^2 - 32x + 24$

$= 3x^3 + 8x^2 - 12x^2 - 6x - 32x + 24$

$= 3x^3 - 4x^2 - 38x + 24$

b) $(5t - 3)(2t^2 - 6t + 12)$ $= 10t^3 - 30t^2 + 60t - 6t^2 + 18t - 36$

$= 10t^3 - 30t^2 - 6t^2 + 60t + 18t - 36$

$= 10t^3 - 36t^2 + 78t - 36$

Example 3:

Simplify the following:

a) $(x + 3)(5x - 2) + 4(x - 1)(2x+5)$

$= 5x^2 - 2x + 15x - 6 + 4(2x^2 + 5x - 2x - 5)$

$= 5x^2 + 13x - 6 + 4(2x^2 + 3x - 5)$

$= 5x^2 + 13x - 6 + 8x^2 + 12x - 20$

$= 5x^2 + 8x^2 + 13x + 12x - 6 - 20$

$= 13x^2 + 25x - 26$

b) $2(3x - 2) - (4x+7)(2x - 5)$

$= 6x - 4 + (-8x^2 + 20x - 14x + 35)$

$= 6x - 4 - 8x^2 + 20x - 14x + 35$

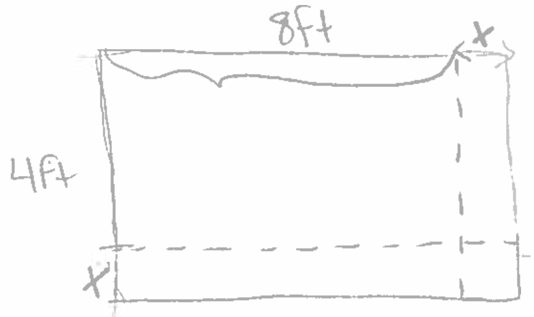
$= -8x^2 + 6x + 20x - 14x - 4 + 35$

$= -8x^2 + 12x + 31$

Example 4:

You are building a skateboard ramp. You have a piece of plywood with dimensions of 4 ft by 8 ft. You cut x ft from the length and the width.

a) Sketch a diagram showing the cuts made to the piece of plywood



$l = 8 - x$

$w = 4 - x$

b) What is the area of the remaining piece of plywood that will be used for the ramp?

$A = lw$

$A = (8-x)(4-x)$

$A = 32 - 8x - 4x + x^2$

$A = 32 - 12x + x^2$

$A = x^2 - 12x + 32$

Key Ideas

- You can use the distributive property to multiply polynomials
  - F.O.I.L (First, Outside, Inside, Last)

  Example:

  $$\begin{aligned}(2x - 5)(x+4) &= (2x)(x) + (2x)(4) + (-5)(x) + (-5)(4) \\ &= 2x^2 + 8x - 5x - 20 \\ &= 2x^2 + 3x - 20\end{aligned}$$

  - Multiply each term in the first polynomial by each term in the second polynomial

  Example:

  $$\begin{aligned}(c-3)(4c^2 - c + 6) &= c(4c^2 - c + 6) - 3(4c^2 - c + 6) \\ &= 4c^3 - c^2 + 6c - 12c^2 + 3c - 18 \\ &= 4c^3 - 13c^2 + 6c - 18\end{aligned}$$

Textbook Questions: Pg. 209 1, 3 - 5, 6(a-d), 7, 10.

5.2 Common Factors

Outcomes: 1. Demonstrate an understanding of factors of whole numbers by determining the:

- Prime factors
- Greatest common factor
- Least common multiple

2. Demonstrate an understanding of common factors and trinomial factoring.

Definitions:

Greatest Common Factor (GCF): the largest factor shared by two or more terms

Example: the GCF of 12 and 42 is 6.

Lowest Common Multiple (LCM): the smallest multiple shared by two or more terms

Example: The multiples of 6 = (6, 12, 18, 24, ...)

The multiples of 3 = (3, 6, 9, 12, 15, 18, ...)

Therefore, the lowest common multiple is 6.

The multiples of 8 = (8, 16, 24, 32, 40, ...)

The multiples of 5 = (5, 10, 15, 20, 25, 30, 35, 40, ...)

Therefore, the lowest common multiple is 40.

Example 1:

Determine the GCF of each pair of terms.

a) 20 and 35

$20 = (1, 2, 4, 5, 10, 20)$

$35 = (1, 5, 7, 35)$

$\therefore GCF = 5$

b) m^2 and m

$m = (1, m)$

$m^2 = (1, m, m^2)$

$\therefore GCF = m$

c) $5m^2n$ *and* $15mn^2$

$5 = (1, 5)$

$15 = (1, 3, 5, 15)$

$m = (1, m)$

$m^2 = (1, m, m^2)$

$n = (1, n)$

$n^2 = (1, n, n^2)$

$\therefore GCF = 5mn$

d) $48ab^3c$ *and* $36a^2b^2c^2$

$48 = (1, 2, 3, 4, 6, 8, 12, 16, 24, 48)$

$36 = (1, 2, 3, 6, 9, 12, 18, 36)$

$\therefore GCF = 12ab^2c$

$a = (1, a)$

$a^2 = (1, a, a^2)$

$c = (1, c)$

$c^2 = (1, c, c^2)$

$b^3 = (1, b, b^2, b^3)$

$b^2 = (1, b, b^2)$

Factored Form: The form of an algebraic expression in which no part of the expression can be made simpler by pulling out a common factor.

Example: The factored form of the algebraic expression 10x + 15 is 5(2x + 3)

Example: The factored form of algebraic expression, $x^2 + 7x + 12$ is (x+4)(x+3).

Example 2:

Write each polynomial in factored form.

a) $4a^2b - 12ab + 8ab^2$ → Take out the GCF (divide by the expression)

GCF = 4ab

$= 4ab(1 \cdot a \cdot 1 - 1 \cdot 3 \cdot 1 + 2 \cdot 1 \cdot b)$

$= 4ab(a - 3 - 2b)$

b) $27r^2s^2 - 18r^3s^2 - 36rs^3$

GCF = $9rs^2$

$= 9rs^2(3r^2 \cdot 1 - 2r^2 \cdot 1 - 4 \cdot 1 \cdot s)$

$= 9rs^2(3r^2 - 2r^2 - 4s)$

Example 3:

Write each expression in factored form.

a) 4(x + 5) - 3x(x + 5)

$= (x+5)(4-3x)$

* this is called grouping

b) $a^2 + 2a + 8ab + 16b$

$= (a^2 + 2a) + (8ab + 16b)$

$= a(a+2) + 8b(a+2)$

$= (a+2)(a+8b)$

Example 4:

The students in Mr. Noyle's Construction class have decided they want to build dog houses for their class project. The class will split up into groups. Each group will construct their dog house with the same type and amount of lumber. Mr. Noyle has 24 ten foot 1 by 4s, 32 eight foot 2 by 4s, and 8 sheets of plywood (4' by 8') available to use for this project.

a) What is the maximum number of groups of students that can build dog houses?

10 ft → 24 → (1, 2, 3, 4, 6, 8, 12, 24)

8ft → 32 → (1, 2, 3, 4, 6, 8, 16, 32) ∴ 8 groups.

plywood → 8 → (1, 2, 4, 8)

b) How much of each lumber type will each group have to work with?

$\frac{24}{8} = 3$ 10 ft. $\frac{32}{8} = 4$ 8ft $\frac{8}{8} = 1$ plywood

c) What is the total length of 2 by 4s and 1 by 4s that each group will have to work with?

$= 3(10ft) + 4(8ft)$

$= 30ft + 32ft$

$= 62ft$

Key Ideas

- Factoring is the reverse of multiplying
- To find the GCF of a polynomial find the GCF of the coefficients and variables.
- To factor a GCF from a polynomial divide each term by the GCF
- Polynomial can be written as a product of the GCF and the sum or difference of the remaining factors.
    - $2m^3n^2 - 8m^2n + 12mn^2 = 2mn(m^2n - 4m + 6n)$
- A common factor can be any polynomial, each as a binomial.
    - $a(x+4) - b(x+4)$ has a common factor of $(x+4)$

Textbook Questions: Pg. 220 - 221 # 1 - 3, 4(c - e), 5(c - e), 6(c - e), 7, 9.

TRY 11, 12, 16

5.3 Factoring Trinomials ($x^2 + bx + c$)

Outcome: Demonstrate an understanding of common factors and trinomial factoring.

Definitions:
Factoring: when two or more binomials are multiplied together, they product a given product. Those two binomials are the factors of the given trinomial.

Example: 30 = 2 x 3 x 5

- The factors of 30 are 2, 3, and 5
- This specific example is also known as prime factorization (*recall from 4.1…*)

Example 1:
Factor the following coefficients:

a) 44 = (1, 44)
(2, 22)
(4, 11)
= (1, 2, 4, 11, 22, 44)

b) 56 = (1, 56)
(2, 28)
(4, 14)
(7, 8)
=(1, 2, 4, 7, 8, 14, 28, 56)

c) 32 = (1, 32)
(2, 16)
(4, 8)
= (1, 2, 4, 8, 16, 32)

** To factor a trinomial of the form $x^2 + bx + c$, first find *two* integers with: **

- A product of c (d x e = c) constant
- A sum of b (d + e = b)

For example, $x^2 + 3x + 2 = (x + 1)(x + 2)$

- 1 x 2 = 2
- 1 + 2 = 3

Example 2:
Factor if possible: $x^2 + 7x + 10$

① What 2 numbers multiply to get 10?
10 = (1, 2, 5, 10)

② What 2 factors of 10 add together to get 7?
1, 10 = 1+10 = 11 X
2, 5 = 2+5 = 7 ✓

$\boxed{= (x+2)(x+5)}$

Recall: $\begin{cases} (+) \times (+) = (+) \\ (-) \times (-) = (+) \\ (-) \times (+) = (-) \end{cases}$

Example 3:

Factor, if possible: $s^2 - 10st + 9t$

① Factors of 9:

$(\pm 1, \pm 3, \pm 9)$

② 2 factors of 9 that add up to -10?

$(-1, -9) = -10$ ☑

$= s^2 - 1st - 9st + 9t$

③ Group

add space

Key Ideas

• To factor a trinomial of the form $x^2 + bx + c$, first find *two* integers with
    ○ A product of c (d x e = c)
    ○ A sum of b (d + e = b)
    ○ For $x^2 + 4x - 12$, find two integers that are
        ■ A product of -12
        ■ A sum of 4
    The two integers are 6 and -2
    Therefore, the factors are $(x + 6)(x - 2)$
• Not all trinomials will factor. Such as $x^2 + 3x + 5$

$= (s^2 - st) + (-9st + 9t)$

$= s(s - t) - 9t(s - t)$

$\boxed{= (s - t)(s - 9t)}$

5.4 Factoring Trinomials ($ax^2 + bx + c$)

Review:

Factor the following.

a) $\cancel{a}^2 - 8x + 16$ x

① Factors of $16 = (\pm 1, \pm 2, \pm 4, \pm 8, \pm 16)$

② 2 factors of 16 that sums up to -8

$(-4 + -4) = -8$

$\boxed{= (x-4)(x-4)}$

b) $y^2 + 5y - 14$

① Factors of $14 = (\pm 1, \pm 2, \pm 7, \pm 14)$

② 2 factors of 14 that sums up to 5

$7 - 2 = 5$

$\boxed{= (y+7)(y-2)}$

Outcome: Demonstrate an understanding of common factors and trinomial factoring.

<u>Factoring a trinomial in the form ax^2+bx+c, with a coefficient in front of a squared variable.</u>

<u>Step 1:</u> Multiply *a* and *c* together.

<u>Step 2:</u> Find two integers with:

- A product of (ac)
- A sum of b

Watch out for what signs the integers have to be.

<u>Step 3:</u> Split b into the two integers that add up to b.

<u>Step 4:</u> Then factor by grouping *(group the first two terms and the last two terms together).*

<u>Example 1:</u>

Factor, if possible.

a) $2x^2 + 7x - 4$

Factors of -8	Product	Sum
$(-1, 8)$	-8	7
$(1, -8)$	-8	-7
$(2, -4)$	-8	-2
$(-2, 4)$	-8	2

$= 2x^2 + 8x - x - 4$

$= 2x(x+4) - x(x+4)$

$\boxed{= (x+4)(2x-x)}$

b) $3b^2 + 24b + 48$

Factors of 144	Product	Sum
(1, 144)	144	145
(2, 72)	144	74
(3, 48)	144	51
(4, 36)	144	40
(6, 24)	144	30
(8, 18)	144	26
(12, 12)	144	24

$= 3b^2 + 12b + 12b + 48$

$= 3b(b+4) + 12(b+4)$

$\boxed{= (b+4)(3b+12)}$

$= (b+4)(3)(b+4)$

$= 3(b+4)(b+4)$

c) $2y^2 + 7xy + 3x^2$

Factors of 6	Product	Sum
(1, 6)	6	7
(2, 3)	6	5

$= 2y^2 + 6xy + xy + 3x^2$

$= 2y(y+3x) + x(y+3x)$

$\boxed{= (y+3x)(2y+x)}$

$4h^2 + 20h + 9$ $\left.\begin{matrix} *36 \\ +20 \end{matrix}\right\} 18, 2$

$(4h^2 + 18h) + (2h+9)$

$= 2h(2h+9) + 1(2h+9)$

$= (2h+1)(2h+9)$

$6x^2 - 21x + 9$

$= 3(2x^2 - 7x + 3)$ $\left.\begin{matrix} *6 \\ +(-7) \end{matrix}\right\} (-6, -1)$

$= 3(2x^2 - 6x - 1x + 3)$

$= 3[(2x^2 - 6x) + (-x+3)]$

$= 3[2x(x-3) - 1(x-3)] = 3(2x-1)(x-3)$

Example 2:

A rescue worker launches a signal flare into the air from the side of a mountain. The height of the flare can be represented by the formula h = $-16t^2 + 144t + 160$. In the formula, h is the height, in feet, above the ground, and t is the time, in seconds.

a) What is the factored form of the formula?

$-16 \times 160 = -2560$

$-16 + 160 = 144$

$= -16t^2 - 16t + 160t + 160$

$= -16t(t+1) + 160(t+1)$

$\boxed{= (t+1)(-16t+160)}$

b) What is the height of the flare after 5.6s?

$h = -16(5.6)^2 + 144(5.6) + 160$

$\boxed{h = 464.64 \text{ ft}}$

Key Ideas

- To factor a trinomial of the form $ax^2 + bx + c$, first factor out the GCF, if possible. Then find two integers with:
  - A product of (a)(c)
  - A sum of b
- Finally, write the middle term as a sum. Then factor by grouping.

  Example:

  $3x^2 - 12x + 6$, the GCF is 3.

  So, $3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

  Identify two integers that:
  - Are a product of (1)(3) = 3
  - A sum of -4

  The two integers are -3 and -1. Use these two integers to write the middle term as a sum. Then factor by grouping.

  $3(x^2 - x - 3x + 3) = 3[x(x-1) - 3(x-1)] = 3(x-3)(x-1)$
- You cannot factor some trinomials, such as $x^2 + 3x + 5$

Textbook Questions: Pg. 234 - 235 #2 - 9

5.5 Factoring Special Trinomials

Outcome: Demonstrate an understanding of common factors and trinomial factoring.

Definitions:

Difference of Squares: an expression of the form $a^2 - b^2$ that involves the subtraction of two squares.

Example:

$(r^2 - 9)$ *which is the same thing as* $(r^2 - 3^2)$

A difference of squares, $a^2 - b^2$, can be factored into $(a + b)(a - b)$

Example 1:

Factor the following binomials.

a) $(x^2 - 16)$

$= (x^2 - \sqrt{16})$

$= (x^2 - 4^2)$

$= (x+4)(x-4)$

b) $(49s^2 - 25)$

$= (\sqrt{49}s^2 - \sqrt{25})$

$= 7^2s^2 - 5^2$

$= (7s)^2 - 5^2$

$= (7s+5)(7s-5)$

c) $(36x^2 - y^2)$

$= (\sqrt{36}x^2 - y^2)$

$= 6^2x^2 - y^2$

$= (6x)^2 - y^2$

$= (6x+y)(6x-y)$

How to identify a difference of squares:

1. The expression is a binomial
2. The first term is a perfect square: x^2
3. The last term is a perfect square: y^2
4. The operation between the two terms is a subtraction

ADDED Questions

1.) $3x^3y^2 - 48x$

$= 3x(x^2y^2 - 16)$

$= 3x((xy+4)(xy-4))$

2) $x^4 - 81$

NOT FULLY FACTORED

$= (x^2+9)\underbrace{(x^2-9)}_{\text{difference of squares}}$

$= (x^2+9)(x-3)(x+3)$

3) $(16 - 0.36y^2)$

$= (4 + 0.6y)(4 - 0.6y)$

When you square a binomial, the result is a perfect square trinomial.

$$
\begin{aligned}
(x + 5)^2 &= (x + 5)(x + 5) \\
&= x(x + 5) + 5(x + 5) \\
&= x^2 + 5x + 5x + 25 \\
&= x^2 + 10x + 25
\end{aligned}
$$

$(x+3)(x+3) = x^2 + \underbrace{6x}_{3+3} + \underbrace{9}_{3\times 3}$

<u>How to identify a perfect square trinomial:</u>

1. The first term is a perfect square: x^2
2. The last term is a perfect square: 5^2
3. The middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(5) = 10x$

<u>Example 2:</u>

Factor the trinomial, if possible.

a) $x^2 + 24x + 144$

① $\sqrt{144} = 12$

② $2 \cdot x \cdot 12 = 24x = b$ ✓

$\boxed{= (x+12)^2}$

b) $y^2 + 18y + 81$

① $\sqrt{81} = 9$

② $2 \cdot y \cdot 9 = 18y = b$

$\boxed{= (x+9)^2}$

<u>Example 3:</u>

Determine two values of *n* that allow each polynomial to be a perfect square trinomial. Then, factor.

a) $y^2 + ny + 36$

① $\sqrt{36} = 6$

② $6 \cdot y \cdot 2 = 12y$

$\boxed{\therefore n = 12}$

b) $5t^2 + nt + 45$

① $\sqrt{5 \cdot 45}$

$= \sqrt{225} = 15$

② $2 \cdot t \cdot 15 = 30t$

$\boxed{\therefore n = 30}$

When you multiply the sum and the difference of two terms, the product will be a difference of squares.

$$\begin{aligned}(x + y)(x - y) &= (x)(x - y) + (y)(x - y)\\ &= (x)(x) - (x)(y) + (y)(x) - (y)(y)\\ &= x^2 - xy + xy - y^2\\ &= x^2 - y^2\end{aligned}$$

Example 4:

Multiply the following factors together. Is the product a difference of squares? Why or why not?

a) $(a + 12)(a - 12)$

$= a^2 - 12a + 12a - 144$

$= a^2 - 144$

$= a^2 - 12^2$

yes, because the middle values cancel out and 144 is a perfect square

b) $(y - 4)(y + 5)$

$= y^2 + 5y - 4y - 20$

$= y^2 + y - 20$

No, because the middle values didnt cancel out

Key Ideas

- Some polynomials are the result of special products. When factoring, you can use the pattern that formed these products.
  - Difference of Squares
    - $x^2 - 36 = x^2 - 6^2$
      $= (x - 5)(x + 5)$
  - Perfect Square Trinomials
    - $x^2 + 14x + 49 = x^2 + 7x + 7x + 49$
      $= x(x + 7) + 7(x + 7)$
      $= (x + 7)(x + 7)$

Textbook Questions: Pg. 246 - 257 # 2 - 8