Name: Key

Chapter 2: Surface Area and Volume

2.1 Surface Area of Prisms and Cylinders

Outcomes: Solve problems, using SI and imperial units, that involve the surface area of 3-D objects, including right prisms and cylinders.

Review:

1) Find the area of a triangle, where the base is 4cm and the height is 10 cm

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(10)$ $\boxed{A = 20cm^2}$

2) Find the area of a rectangle where the length is 12 m and the width is 7 m

$A = lw$

$A = (12)(7)$

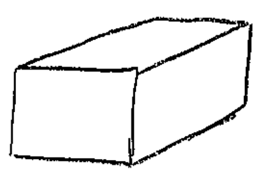
$A = 84m^2$

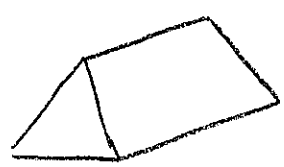
Definitions:

Surface Area: the sum of the areas of each face of the object.

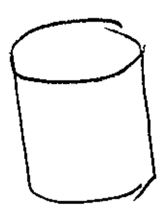
ALWAYS MEASURED IN SQUARE UNITS

Prisms: three-dimensional object which has two bases or ends that have the same size and shape and are parallel to each other.



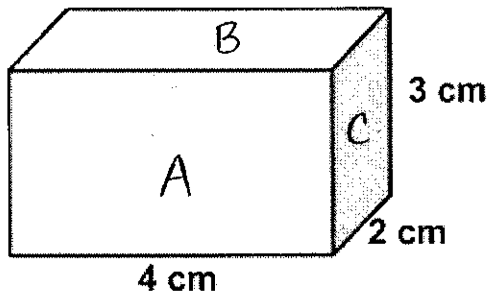


Cylinder: Three-dimensional object with two parallel and congruent circular bases.



Example 1:

Identify what three-dimensional shape is and the surface area of the shape below.



$SA = 2A + 2B + 2C$

$SA = 2(4\times3) + 2(4\times2) + 2(3\times2)$

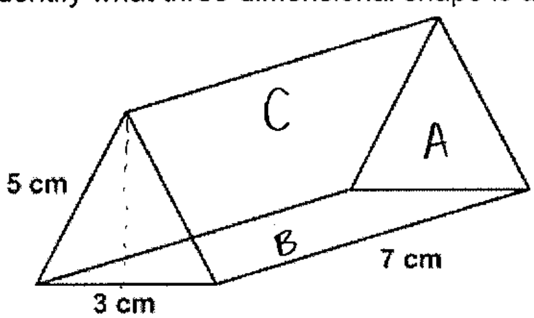
$SA = 24 + 16 + 12$

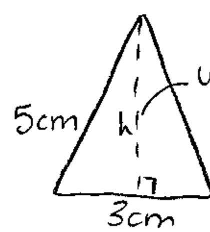
$SA = \boxed{52\,cm^2}$

units are squared!

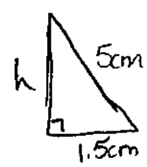
Example 2:

Identify what three-dimensional shape is and the surface area of the shape below.





we need to solve for height of the triangle!



$h^2 + b^2 = c^2$

$h^2 + 1.5^2 = 5^2$

$h^2 + 2.25 = 25$

$\quad -2.25 \quad -2.25$

$\sqrt{h^2} = \sqrt{22.75}$

$\boxed{h = 4.77\,cm}$

$SA = 2A + B + 2C$

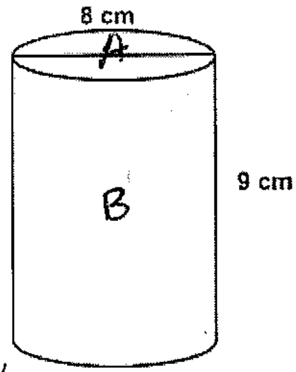
$SA = 2\left(\frac{3\times4.77}{2}\right) + (3\times7) + 2(5\times7)$

$SA = 14.31 + 21 + 70$

$\boxed{SA = 105.31\,cm^2}$

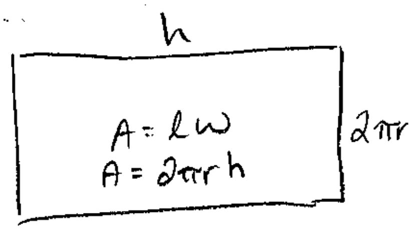
Example 3:

Identify what three-dimensional shape is and the surface area of the shape below.



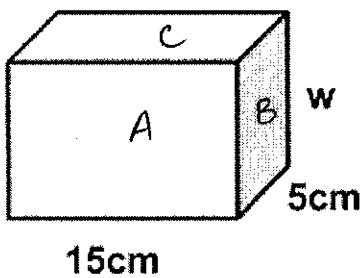
* roll their paper into the shape of a cylinder, what shapes make a cylinder?

$$SA = 2A + B$$
$$= 2(\pi r^2) + (2\pi r h)$$
$$= 2(\pi(4)^2) + (2\pi(4)(9))$$
$$= 326.73\text{ cm}^2$$



Example 4:

Find the width if the SA = 500cm^2



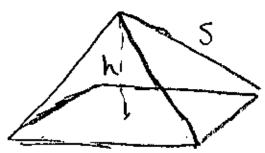
$$SA = 2A + 2B + 2C$$
$$500 = 2(15 \times w) + 2(5 \times w) + 2(15 \times 5)$$
$$500 = 30w + 10w + 150$$
$$-150 \qquad -150$$
$$\frac{350}{40} = \frac{40w}{40}$$
$$\boxed{8.75\text{cm} = w}$$

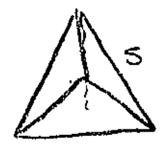
Textbook Questions: Pg. 74 # 1ab, 3ab, 4, 6, 9, 11, 17

2.2 Surface Area of Pyramids and Cones

Outcomes: Solve problems, using SI and imperial units, that involve the surface area of 3-D objects, including right pyramids and cones.

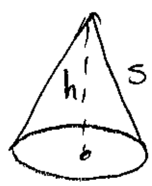
Definitions:
Pyramid: A three-dimensional object with one base and the same number of triangular faces as there are sides on the base.





For example: Square-based Pyramid; Rectangular Pyramid; Triangular Pyramid

Cone: Three-dimensional object with a circular base and a curved lateral side that extends from the base to the vertex.



Slant Height: the shortest lateral distance from the edge of the base of a pyramid to its highest point.

Lateral Area: the surface that joins the two bases of a three-dimensional object or that joins the base to the highest point

Review:
Find the area of a circle with:

a) Diameter of 6ft → $d = 2r$ $\frac{6}{2} = \frac{2r}{2}$ $r = 3$

$A = \pi r^2$

$A = \pi (3)^2$ $= \boxed{28.27 ft^2}$

b) Radius of 2cm

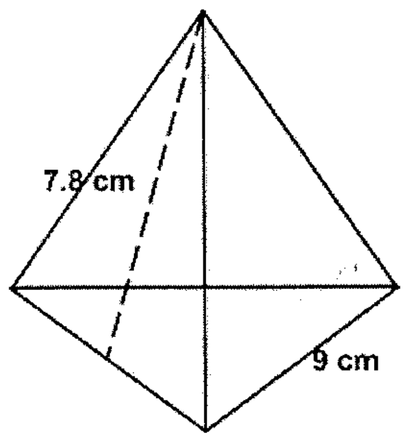
$A = \pi r^2$

$A = \pi (2)^2$

$\boxed{A = 12.566 cm^2}$

Example 1:

Calculate the surface area of the tetrahedron pyramid.



$\boxed{\text{tetrahedron = all same sides}}$

$SA = 4(\text{side})$

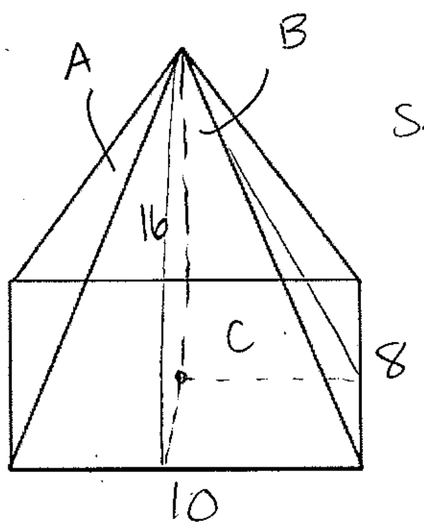
$= 4\left(\frac{7.8 \times 9}{2}\right)$

$= \boxed{140.4\,cm^2}$

Example 2:

A right rectangular pyramid has base dimensions 8ft by 10ft, and a height of 16ft. Calculate the surface area of the pyramid to the nearest square foot.

height from the middle of the base to the apex... <u>NOT</u> the slant height!



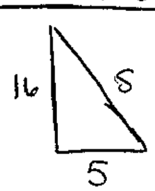
$SA = 2A + 2B + C$

$SA = \cancel{2}\left(\frac{8 \times 16.763}{\cancel{2}}\right) + \cancel{2}\left(\frac{10 \times 16.4924}{\cancel{2}}\right) + (10 \times 8)$

$SA = 134.104 + 164.924 + 80$

$\boxed{SA = 379\,ft^2}$

(A) 1 <u>Side Face</u>

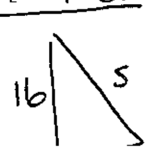


$a^2 + b^2 = s^2$

$5^2 + 16^2 = s^2$

$16.763... = s$

(B) <u>Front Face</u>

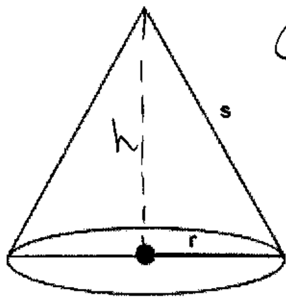


$a^2 + b^2 = s^2$

$4^2 + 16^2 = s^2$

$16.11924... = s$

Surface Area of a Cone:



$SA = \pi rs + \pi r^2$

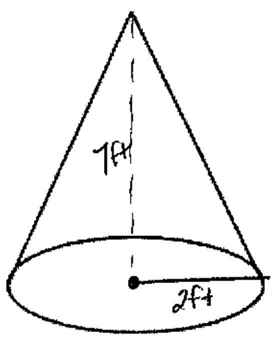
lateral Area

Area of Base

Example 3:

not slant height

A right cone has a base radius of 2ft and a height of 7ft. Calculate the surface area of the cone to the nearest foot.



* we need slant height *

$a^2 + b^2 = s^2$

$2^2 + 7^2 = s^2$

$7.3ft = s$

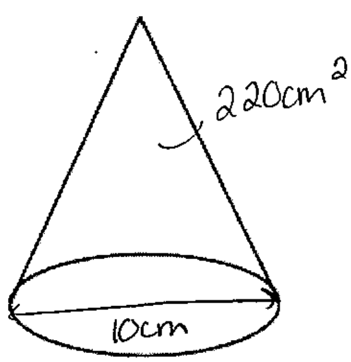
$SA = \pi(2)(7.3) + \pi(2)^2$

$SA = 58.3ft^2$

$\boxed{= 58 ft^2}$

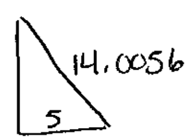
Example 4:

The lateral area of a cone is 220 cm^2. The diameter of the cone is 10 cm. Determine the height of the cone, to the nearest tenth of a cm.



$SA = \pi rs + \pi r^2$

L.A



① $L.A = \pi rs$

$\frac{220cm^2}{5\pi} = \frac{\pi(5)s}{5(\pi)}$

$14.0056 cm = s$

② $a^2 + b^2 = c^2$

$a^2 + 5^2 = 14.0056^2$

$a^2 = 14.0056^2 - 5^2$

$a = 13.08$

$\boxed{h = 13.1cm}$

Textbook Questions: Pg. 74. #1cd, 2, 7, 12, 14, 16

2.3 Surface Area Spheres

Outcomes: Solve problems, using SI and imperial units, that involve the surface area of 3-D objects, including spheres

Definitions:

<u>Sphere:</u> a round, ball-shaped object

A set of points in space that are a given distance (radius) from a fixed point (centre)

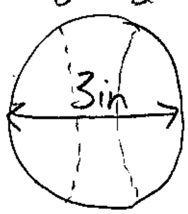
Surface Area of a Sphere:

$$SA = 4\pi r^2$$

Example 1:

The diameter of a baseball is approximately 3 in. Determine the surface area of a baseball to the nearest square inch.

$r = \frac{d}{2} = \frac{3}{2} = 1.5in$



$SA = 4\pi r^2$

$SA = 4\pi(1.5)^2$

$= 28.2743$

$\boxed{= 28 in^2}$

Example 2:

The surface area of a lacrosse ball is approximately 20 square inches. What is the diameter of the lacrosse ball to the nearest tenth of an inch.

$SA = 4\pi r$

$\frac{20in^2}{4\pi} = \frac{\cancel{4\pi} r^2}{\cancel{4\pi}}$

$\sqrt{\frac{20in^2}{4\pi}} = \sqrt{r^2}$

$r = 1.2615...$

$d = 1.2615 \times 2$

$\boxed{d = 2.5in}$

Textbook Questions: Pg. 74 #1e, 3c, 5, 13, 15

2.4 Volume of Prisms, Cylinders and Spheres

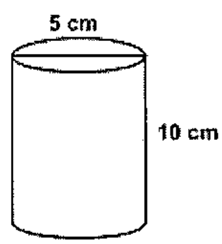
Outcomes: Solve problems, using SI and imperial units, that involve the volume of 3-D objects, including right prisms, right cylinders, and spheres.

To calculate the volume of prisms and cylinders, simply calculate the area of the base then multiply by the height.

Example 1:

1. Calculate the volume of each shape:

a)



$V = Bh = \pi r^2 h$

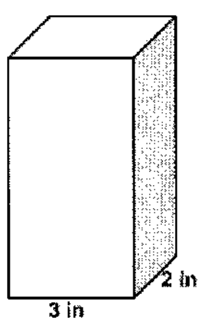
↑ base is circular!

$= \pi(2.5)^2(10)$

$= 196.3495 \text{ cm}$

$\boxed{= 196.35 \text{ cm}^3}$

b)



$V = Bh = lwh$

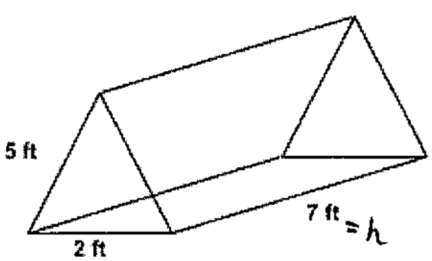
↗ base is rectangular

$= 3 \times 2 \times 5$

$= 30 \text{ in}^3$

* don't assume the base is what ays flat on the ground *

c)



$V = Bh = \left(\frac{bh}{2}\right)h$

↗ base is triangular

$= \left(\frac{2 \times 4.8989}{2}\right) \times 7$

$= 4.8989 \times 7$

$\boxed{= 34.29 \text{ ft}^3}$

$a^2 + h^2 = c^2$

$1^2 + h^2 = 5^2$

$1^2 + h^2 = 25$

$h^2 = 25 - 1$

$\sqrt{h^2} = \sqrt{24}$

$h = 4.8989\ldots$

To calculate the volume of a sphere, use the formula $V = \frac{4}{3}\pi r^3$

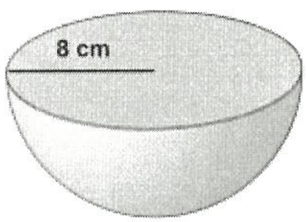
2. Calculate the volume of the sphere that has a radius of 4.5 inches.

$$V = \frac{4}{3}\pi (4.5)^3$$

$$\boxed{= 381.7 in^3}$$

3. A hemisphere has a radius of 8 cm.

a) What is the surface area of the hemisphere to the nearest tenth of a square cm?



$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$= 2\pi r^2 + \pi r^2$$

$$= 2\pi (8)^2 + \pi (8)^2$$

$$= 603.1857...$$

$$\boxed{= 603.2 cm^3}$$

b) What is the volume of the hemisphere to the nearest tenth of a cubic cm?

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{1}{2}\left(\frac{4}{3}\pi (8)^3\right)$$

$$= 1072.3302...$$

$$\boxed{= 1072.3 cm^3}$$

Textbook Questions: Pg. 86 #1(bce), 3, 5(ac), 6-8, 13, 14

2.5 Volume of Pyramids and Cones

Outcomes: Solve problems, using SI and imperial units, that involve the volume of 3-D objects, including right pyramids and cones.

To calculate the volume of pyramids, use the following formula:

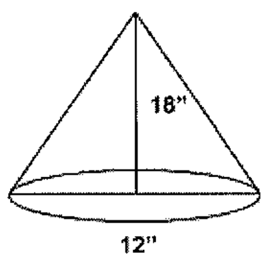
$$V = \frac{1}{3}lwh$$

To calculate the volume of cones, use the following formula:

$$V = \frac{1}{3}\pi r^2 h$$

1. Calculate the volume of each shape:

a)

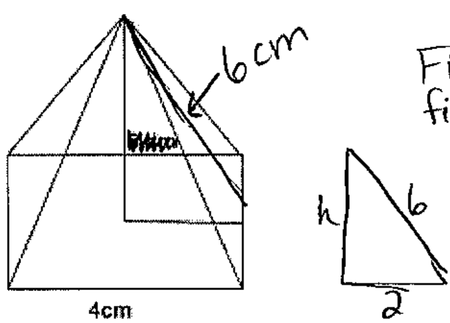


$V = \frac{1}{3}\pi(6)^2(18)$

$= 678.584\ldots$

$\boxed{= 679 \text{ in}^3}$

b)



First need to find the height!

$h^2 + 2^2 = 6^2$

$h^2 = 36 - 4$

$\sqrt{h^2} = \sqrt{32}$

$h = 5.65685\ldots$

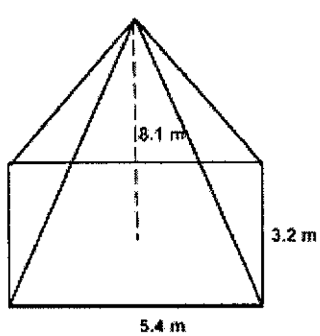
$V = \frac{1}{3}lwh$

$= \frac{1}{3}(4)(4)(5.65685)$

$= 30.1699 \text{ cm}^3$

$\boxed{V = 30.2 \text{cm}^3}$

c)



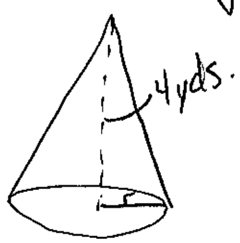
$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(5.4)(3.2)(8.1)$

$\boxed{V = 46.656 \text{m}^3}$

2. A cone has a height of 4 yd and a volume of 205 cubic yards. Determine the radius of the base of the cone to the nearest whole number.

$V = 205 \, yd^3$



$$V = \frac{1}{3}\pi r^2 h$$

$$205 = \frac{1}{3}\pi r^2 (4)$$

$$\frac{205}{(\frac{4}{3}\pi)} = \frac{\frac{4}{3}\pi r^2}{(4/3\pi)}$$

$$\sqrt{48.940145} = \sqrt{r^2}$$

$$6.9957 \, yd = r$$

$$\boxed{7 \, yd = r}$$

Textbook Questions: Pg. 86 # 1(ab), 2(ab), 9 - 11, 12, 18

OR

$$V = \frac{1}{3}\pi r^2 h$$

$$3 \times (205) = (\frac{1}{3}\pi r^2 (4)) \times 3$$

$$\frac{615}{4} = \frac{4\pi r^2}{4}$$

$$\frac{153.75}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{48.940145} = \sqrt{r^2}$$

$$\boxed{7 \, yd = r}$$