Name: Key

Chapter 9 Solving Systems of Linear Equations Algebraically

9.1 Solving Systems of Linear Equations by Substitution

Outcomes: 1. Interpret algebraic reasoning through the study of relations
9. Solve problems that involve systems of linear equations in two variables algebraically.

Definitions:
Substitution Method: and algebraic method of solving a system of equations.

- Solving one equation for one variable, substitute that value into the other equation, and solve for the other variable.

Steps for Solving by Substitution
1) Isolate one variable in one of the equations 2) Substitute that solution into the other equation, and solve for the remaining variable 3) Substitute the value into one of the original equation to solve for the other variable 4) Check your answer by substituting into BOTH original equations.

Example 1:
Solve the following linear system algebraically using substitution.

a) $3x + 5y = 27$
$4x = 16$

1) $\frac{4x}{4} = \frac{16}{4}$ $x = 4$

2) $3(4) + 5y = 27$

$12 + 5y = 27$
$-12 \qquad -12$

$\frac{5y}{5} = \frac{15}{5}$

$y = 3$

$\therefore$ (4,3) is the solution

4) Check

$3x+5y=27$	
$3(4)+5(3)$	27
$12+15$	27
27	27 ✓

$4x$	$=16$
$4(4)$	16
16	16 ✓

b) 4x + 5y = 26
3x = y - 9

1) $\frac{3x}{3} = \frac{y}{3} - \frac{9}{3} \rightarrow x = \frac{y}{3} - 3$

2) $4(\frac{1}{3}y - 3) + 5y = 26$

$\frac{4}{3}y - 12 + 5y = 26$
$\quad +12 \qquad\qquad +12$

$\frac{4}{3}y + 5y = 38$

$\frac{4}{3}y + \frac{15}{3}y = 38$

$\frac{\frac{19}{3}y}{(\frac{19}{3})} = \frac{38}{(\frac{19}{3})}$

$y = 6$

3) $3x = 6 - 9$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

4) $4x + 5y = 26$
$4(-1) + 5(6) = 26$
$-4 + 30 = 26$
$26 = 26$ ✓

$3x = y - 9$
$3(-1) = 6 - 9$
$-3 = -3$ ✓

$\therefore (-1, 6)$ is a solution

Example 2:
Solve the following linear system algebraically using substitution. Check your solution.
2x + y = 13
x - 0.4y = -16

1) $2x + y = 13$

$y = -2x + 13$

2) $x - 0.4(-2x + 13) = -16$

$1x + 0.8x - 5.2 = -16$

$1.8x - 5.2 = -16$
$\quad +5.2 \quad +5.2$

$\frac{1.8x}{1.8} = \frac{-10.8}{1.8}$

$x = -6$

3) $2(-6) + y = 13$

$-12 + y = 13$
$+12 \qquad +12$

$y = 25$

4) $2x + y = 13$
$2(-6) + 25 = 13$
$-12 + 25 = 13$
$13 = 13$ ✓

$-6 - 0.4(25) = -16$
$-6 - 10 = -16$
$-16 = -16$

$\therefore (-6, 25)$ is a solution

Example 3:

At a basketball game, there were 220 people. Tickets cost \$9.00 for adults, and \$6.00 for children. The school collected \$1614.00 in ticket sales. How many adults and how many children attended the basketball game?

let a = adult
c = children

$a + c = 220$

$9a + 6c = 1614$

1) $a = 220 - c$

2) $9(220-c) + 6c = 1614$
$1980 - 9c + 6c = 1614$
$1980 - 3c = 1614$
$-1980 \quad -1980$
$-3c = -366$
$\frac{-3c}{-3} = \frac{-366}{-3}$
$c = 122$

3) $a + 122 = 220$
$-122 \quad -122$
$a = 98$

$\boxed{(122, 98)}$

∴ there are 122 children an[d] 98 adults

Key Ideas

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations
  - Where possible, choose a variable with a coefficient of 1.

    Solve the linear system

    $3x + 2y = -11$ (a)

    $-2x + y = 12$ (b)

    Isolate the variable y in equation (b) since its coefficient is 1.

    $y = 12 + 2x$

    Substitute the expression for y in (a)

    $3x + 2(12 + 2x) = -11$

    $3x + 24 + 4x = -11$

    $7x + 24 = -11$

    $7x = -35$

    $x = -5$
  - Substitute the solution for the first variable into the one of the original equations. Solve for the remaining variable.

    $-2(-5) + y = 12$

    $10 + y = 12$

    $y = 2$
  - Check you answer by substituting into **BOTH** original equations.

Textbook Questions: pg. 474 # 1 - 4, 6-14, 16, 19

$3x + 2y = 11$ → solve by Substitution
$-x + y = 3$
→ show elimination for x, & y.

9.2 Solving Systems of Linear Equations by Elimination

Outcomes: 1. Interpret algebraic reasoning through the study of relations
9. Solve problems that involve systems of linear equations in two variables algebraically.

Definitions:
Elimination Method: an algebraic method of solving a system of equations.
- Add or subtract the equations to eliminate one variable and solve for the other variable

Steps for Solving by Elimination
1) Make sure the coefficients of one variable are the same 2) Add or subtract equations to eliminate one variable 3) Substitute the value into one of the original equation to solve for the other variable 4) Check your answer by substituting into BOTH original equations.

Example 1:
Solve the system of linear equations algebraically by elimination.
a) $2x + 3y = 20$
$2x + y = 4$

1) 2x are the same in both

2)
$$\begin{array}{r} 2x + 3y = 20 \\ -\quad 2x + y = 4 \\ \hline 0 + 2y = 16 \end{array}$$

$$\frac{2y}{2} = \frac{16}{2} \rightarrow y = 8$$

3)
$$2x + 8 = 4$$
$$-8 \quad -8$$
$$\frac{2x}{2} = \frac{-4}{2}$$
$$x = -2$$

4)
$$2(-2) + 3(8) = 20$$
$$-4 + 24 = 20$$
$$20 = 20 \checkmark$$

$$2(-2) + 8 = 4$$
$$-4 + 8 = 4$$
$$4 = 4 \checkmark$$

∴ (-2, 8) is a solution

b) $3x - 4y = 7$
$5x - 6y = 8$

1) $5(3x-4y=7) \rightarrow$
$3(5x-6y=8)$

2)
$15x - 20y = 35$
$-\ 15x - 18y = 24$
$0 - 2y = 11$

$\frac{-2y}{-2} = \frac{11}{-2}$ $y = -5.5$

3) $3x - 4(-5.5) = 7$
$3x + 22 = 7$
$-22 \quad -22$
$\frac{3x}{3} = \frac{-15}{3}$ $x = -5$

4) $3(-5) - 4(-5.5) = 7$
$-15 + 22 = 7$
$7 = 7$ ✓
$5(-5) - 6(-5.5) = 8$
$-25 + 33 = 8$
$8 = 8$ ✓
$\therefore (-5, -5.5)$ is a solution

Example 2:

Solve the system of linear equations algebraically by elimination. Check your answers.

1) $3(2x + 7y = 24)$
$2(3x - 2y = -4)$

2) $6x + 21y = 72$
$-\ 6x - 4y = -8$
$0 + \frac{25y}{25} = \frac{80}{25}$
$y = 3.2$

3) $2x + 7(3.2) = 24$
$2x + 22.4 = 24$
$-22.4 \quad -22.4$
$\frac{2x}{2} = \frac{1.6}{2}$ $x = 0.8$

4) $2(0.8) + 7(3.2) = 24$
$1.6 + 22.4 = 24$
$24 = 24$ ✓

$3(0.8) - 2(3.2) = -4$
$2.4 - 6.4 = -4$
$-4 = -4$ ✓

$\therefore (0.8, 3.2)$ is a solution

Example 3:

A group of people bought tickets for a University of Alberta football playoff game. Two students tickets and six adult tickets cost $102. Eight student tickets and three adult tickets cost $114. What was the price for a single adult ticket? What was the price for a single student ticket?

let s= students
a=adult

1) $4(2s+6a=102)$
$8s+3a=114$

2) $8s+24a=408$
$-\ 8s+3a=114$
$21a=294$
$\frac{21a}{21}=\frac{294}{21}$
$a=14$

3) $2s+6(14)=102$
$2s+84=102$
$-84\quad -84$
$\frac{2s}{2}=\frac{18}{2}$
$s=9$

4) $2(9)+6(14)=102$
$18+84=102$
$102=102$ ✓

$8s+24a=408$
$8(9)+24(14)=408$
$72+336=408$
$408=408$ ✓

∴ there are 9 students and 14 adults

Example 4:

try on their own

During lunch, the cafeteria sold a total of 160 muffins and individual yogurts. The price of each muffin is $1.50. Each container of yogurt costs $2.00. The cafeteria collected $273.50. Set up and solve a linear system in order to determine the number of muffins and the number of yogurts sold.

Let m= muffin
y= yogurt

$2(m+y=160)$
$1.5m+2y=273.50$

$2m+2y=320$
$-\ 1.5m+2y=273.50$
$0.5m=46.50$
$\frac{0.5m}{0.5}=\frac{46.50}{0.5}$
$m=93$

∴ they sold 93 muffins and 67 yogurts.

3) $m+y=160$
$93+y=160$
$-93\quad -93$
$y=67$

4) $m+y=160$
$93+67=160$
$160=160$ ✓

$1.5(93)+2(67)=273.50$
$139.5+134=273.50$
$273.50=273.50$ ✓

Key Ideas

- A table can help you organize information in a problem. This can help you to determine the equations in a linear system
- You can solve a linear system by elimination.

  $3x + 2y + 6 = 0$

  $7y = 5x + 41$

  - If necessary, rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$

    $3x + 2y + 6 = 0$ $7y = 5x + 41$

    $3x + 2y = -6$ (a) $-5x + 7y = 41$ (b)

  - Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

    Multiply (a) by 5 and (b) by 3 so that the coefficients of the terms involving x add to zero

    $5(3x + 2y) = 5(-6)$ $3(-5x + 7y) = 3(41)$

    $15x + 10y = -30$ $-15x + 21y = 123$

    Add to eliminate x

    $$\begin{array}{r} 15x + 10y = -30 \\ \underline{+\ (-15x + 21y = 123)} \\ 31y = 93 \end{array}$$

  - Solve for the remaining variable.

    $31y = 93$

    $y = 3$

  - Solve for the second variable by substituting the value for the first variable into one of the original equations.

    $7(3) = 5x + 41$

    $21 = 5x + 41$

    $-20 = 5x$

    $-4 = x$

  - Check your solution by substituting each value into **BOTH** original equations.

Textbook Questions: Pg. 488 # 1-13, 15

9.3 Solving Problems Using Systems of Linear Equations

Outcomes: 1. Interpret algebraic and graphical reasoning through the study of relations
3. Demonstrate an understanding of slope with respect to:
- Rise and run
- Line segments and lines
- Rate of change
- Parallel lines
- Perpendicular lines

7. Determine the equations of a linear relation, given:
- A graph
- A point and the slope
- Two points
- A point and the equation of a parallel or perpendicular line

to solve problems.
9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Overall, you can solve:
Graphically
- Time consuming
- Provides visual as how questions are related

OR

Algebraically
- Allows for exact solution relatively quickly
- Easy to make errors

Example 1:

Solve the linear system twice, using both algebraic methods. Compare the two methods.

1) $4(3x - 4y = 17)$
$3(4x + 5y = 48.5)$

Elimination

2) $12x - 16y = 68$
$- \; 12x + 15y = 145.5$
$-31y = -77.5$
$\frac{-31y}{-31} = \frac{-77.5}{-31}$
$y = 2.5$

3) $3x - 4(2.5) = 17$
$3x - 10 = 17$
$+10 \quad +10$
$\frac{3x}{3} = \frac{27}{3}$
$x = 9$

Check

4) $3x - 4y = 17$
$3(9) - 4(2.5) = 17$
$27 - 10 = 17$
$17 = 17$ ✓
$4(9) + 5(2.5) = 48.5$
$36 + 12.5 = 48.5$
$48.5 = 48.5$ ✓

Substitution

1) $3x - 4y = 17$
$+4y \quad +4y$
$\frac{3x}{3} = \frac{17}{3} + \frac{4y}{3}$
$x = \frac{17}{3} + \frac{4}{3}y$

2) $4\left(\frac{17}{3} + \frac{4}{3}y\right) + 5y = 48.5$
$\frac{68}{3} + \frac{16}{3}y + 5y = 48.5 - 68/3$
$-68/3$
$\frac{16}{3}y + \frac{15}{3}y = \frac{145.5}{3} - \frac{68}{3}$
$\frac{31}{3}y / (3^1/_3) = \frac{77.5}{3} / (3^1/_3)$
$y = 2.5$

3) $4x + 5(2.5) = 48.5$
$4x + 12.5 = 48.5$
$-12.5 \quad -12.5$
$\frac{4x}{4} = \frac{36}{4}$
$x = 9$

$\therefore$ (9, 2.5) is a solution

Example 2:

At Costco, 2 poutines and 3 hot dogs cost \$13.50. 5 poutines and 4 hot dogs cost \$21.50. What is the price of an individual hot dog and poutine?

p = poutines

h = hot dogs

1) $5(2p + 3h = 13.50)$

$2(5p + 4h = 21.50)$

2) $10p + 15h = 67.5$

$-\ 10p + 8h = 43$

$7h = 24.5$

$\frac{7h}{7} = \frac{24.5}{7}$

$h = 3.5$

3) $2p + 3(3.5) = 13.50$

$2p + 10.50 = 13.50$

$-10.5 \quad -10.5$

$\frac{2p}{2} = \frac{3}{2}$ $p = 1.5$

4) $2p + 3h = 13.50$

$2(1.5) + 3(3.5) = 13.50$

$3 + 10.50 = 13.50$

$13.50 = 13.50$ ✓

∴ A hot dog costs \$3.50 and a poutine costs \$1.50

Example 3:

The percentage of carbohydrates by mass in an orange is 15%. The percentage of carbohydrates by mass in an apple is 25%. Bill eats 175g of carbohydrates in a mixture of apples and oranges that contains 19.3% carbohydrates. How many grams of apples did he eat? How many grams of oranges?

a = apples (g)

x = oranges (g)

1) $x + a = 175$

$0.15x + 0.25a = 33.775$

the total needs to be a weight since x and a are weights

2) $x = 175 - a$

$0.15(175 - a) + 0.25a = 33.775$

$26.25 - 0.15a + 0.25a = 33.775$

$26.25 + 0.10a = 33.775$

$\frac{0.10a}{0.10} = \frac{7.525}{0.10}$

$a = 75.25g$

3) $x + 75.25 = 175$

$-75.25 \quad -75.25$

$x = 99.75$

∴ there is 75.25 grams of apples and 99.75 grams of oranges

Example 4:

The rectangular parking pad for a car has a perimeter of 12.2m. The width is 0.7m shorter than the length. Use a linear system to determine the dimensions of the parking pad.

l = length

w = width

$2l + 2w = 12.2$

$w = l - 0.7$ ← substitute!

2) $2l + 2(l - 0.7) = 12.2$

$2l + 2l - 1.4 = 12.2$

$+1.4 \quad +1.4$

$\frac{4l}{4} = \frac{13.6}{4}$

$l = 3.4m$

3) $w = l - 0.7$

$w = 3.4 - 0.7$

$w = 2.7m$

4) $2l + 2w = 12.2$

$2(3.4) + 2(2.7) = 12.2$

$12.2 = 12.2$ ✓

$w = l - 0.7$

$2.7 = 3.4 - 0.7$

$2.7 = 2.7$ ✓

∴ the length is 3.4m and the width is 2.7m

Key Ideas

- System of linear equations can be solved:
  - Graphically
  - Algebraically by substitution or by elimination
- It may be better to use a graphical approach to solve linear systems when you wish to see how the two variables relate, such as for cost analysis and speed problems
- It may be better to use an algebraic approach to solve linear equations when:
  - You need only the solution (point of intersection)
  - It is unclear where to locate the solution on a coordinate plane

Textbook Questions: Pg. 498 # 1 - 3, 5 - 12