Name: Key

Chapter 8 Solving Systems of Linear Equations Graphically

8.1 System of Linear Equations and Graphs

Outcome: 1. Interpret graphical reasoning through the study of relations

3. Demonstrate an understanding of slope with respect to:
- Rise and run
- Line segments and lines
- Parallel lines
- Perpendicular lines

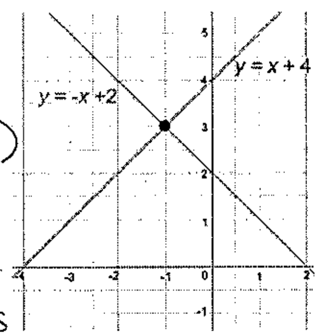
9. Solve problems that involve systems of linear equations in two variables graphically

Definitions:

Point of Intersection: a point at which two lines touch or cross

*****HAVE ~~YOU~~ to VERIFY THE POINT IN BOTH EQUATIONS*****

Determine the point of intersection on the graph. (-1, 3)



Point of Intersection on Calc.

Graph → 2^{nd} → Trace → Intersect → Enter 3 times

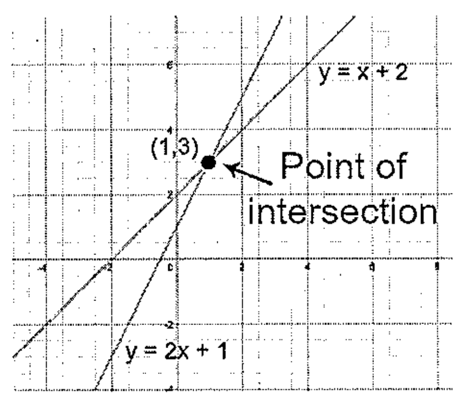
System of Linear Equations: two or more linear equations involving common variables.

Example: $-2x + y = 3$ and $4x - 11 = y$

Non-example: $\frac{1}{2}x - y = 5$ and $2s + 1 = t$

Solution (to a system of linear equations): a point of intersection of the lines on a graph
- An ordered pair that satisfies both equations

Verify that (1, 3) satisfies $y = x + 2$ and $y = 2x + 1$:



y	$= x + 2$
3	$1 + 2$
3	3 ✓

y	$= 2x + 1$
3	$2(1) + 1$
3	$2 + 1$
3	3 ✓

(1, 3) is a solution

- A pair of values occurring in the table of values of both equations

x + y = 5	
0	5
1	4
2	3
3	2
4	1

2x + y = 8	
0	8
1	6
2	4
3	2
4	0

* look for the same point in both tables

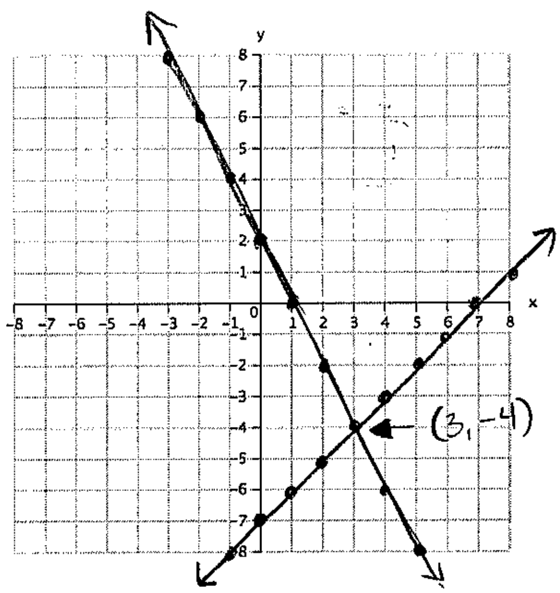
(3, 2) is the point of intersection

Example 1:
Graph the two linear equations, 2x + y = 2 and x - y = 7, on paper and on your calculator.

$$2x + y = 2$$
$$-2x \qquad -2x$$
$$y = -2x + 2$$
slope y-inter

$$x - y = 7$$
$$+y \qquad +y$$
$$x = 7 + y$$
$$-7 \qquad -7$$
$$y = 1x - 7$$
slope y-inter

a) Identify the point of intersection



b) Verify the solution

$2x+y$	$= 2$
$2(3)-4$	2
$6-4$	2
2	2 ✓

$x-y$	$= 7$
$3-(-4)$	7
$3+4$	7
7	7 ✓

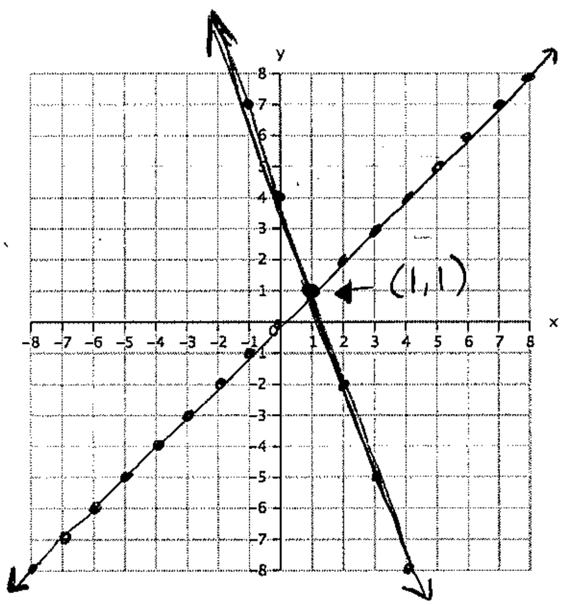
* TIP
Use the original equation given to verify!

(3, -4) is a solution

Example 2:

a) Consider the system of linear equations 3x + y = 4 and x - y = 0. Identify the point of intersection of the lines by graphing.

$y = -3x + 4$ $\quad$ $y = x$



Verify the solution.

$3x+y$	$= 4$
$3(1)+1$	4
$3+1$	4
4	4 ✓

The point of intersection is (1,1)

$x-y$	$= 0$
$1-1$	0
0	0 ✓

Example 3:

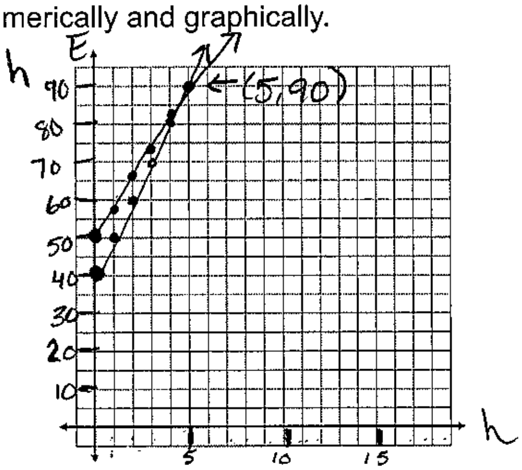
Davidee earns \$40 plus \$10 per hour. Carmen earns \$50 plus \$8 per hour.

a) Represent the linear system relating the earnings numerically and graphically.

D: $E_D = 40 + 10h$ $\quad$ C: $E_C = 50 + 8h$

h	E_D
0	40
1	50
2	60
3	70
4	80
5	90.

h	E_C
0	50
1	58
2	66
3	74
4	82
5	90



b) Identify the solution to the linear system and explain what it represents.

(5, 90) is a point of intersection, it represents that when Davidee and Carenen work 5 hrs the both earn \$90

VERIFY

E_D	$40+10h$
90	$40+10(5)$
90	$40+50$
90	90 ✓

E_C	$50+8h$
90	$50+8(5)$
90	$50+40$
90	90 ✓

Example 4:

For each system of linear equations, verify whether the given point is a solution.

a) $x + 4y = 22$

$3x - y = 2$

(2, 5)

b) $2x + 3y = -12$

$4x - 3y = -6$

(-3, -2)

∴ a solution

$x + 4y$	$= 22$
$2+4(5)$	22
$2+20$	22
22	22 ✓

$2x+3y$	$= -12$
$2(-3)+3(-2)$	-12
$-6+-6$	-12
-12	-12 ✓

∴ Not a solution

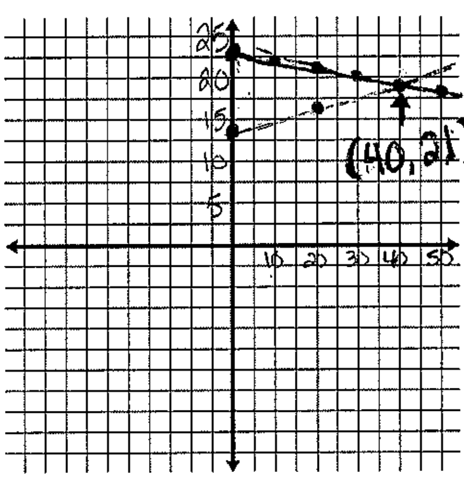
$3x - y$	$= 2$
$3(2)-5$	2
$6-5$	2
1	1 X

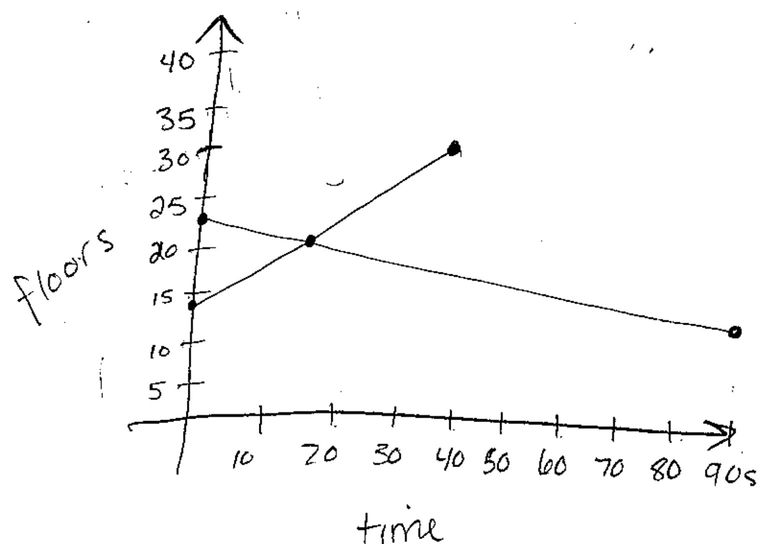
$4x - 3y$	$= -6$
$4(-3)-3(-2)$	-6
$-12-(-6)$	-6
$-12+6$	-6
-6	-6 ✓

Example 5:

Eric works on the 23rd floor of a building. It takes Eric 90s to walk down the stairs to the 14th floor. Nathan works on the 14th floor and needs to go up to the 30th floor. He knows it will take 40s by elevator if the elevator makes no other stops.

Suppose both men leave their offices at the same time. Create a graph to model their travel.





b) What does the point of intersection represent?

represents it'll take 40s to reach floor 21 at the same time.

Key Ideas

- Systems of linear equations can be modelled numerically, graphically, or algebraically.
- The solution to a linear system is a pair of values that occurs in each table of values, intersection point of the lines, or an ordered pair that satisfies each equation.
- A solution to a system of linear equations can be verified using several methods:
    - Substitute the value for each variable and evaluate the equation
    - Create a graph and identify the point of intersection
    - Create table of values and identify the pair of values that occurs in each table

Textbook Questions: Pg. 427 #1 - 15, $\underbrace{17, 19, 20}_{\text{extra.}}$

8.2 Modelling and Solving Linear Systems

Outcome: 1. Interpret graphical reasoning through the study of relations
7. Determine the equations of a linear relation, given:
- A graph
- A point and the slope
- Two points
- A point and the equation of a parallel or perpendicular line

to solve problems.
9. Solve problems that involve systems of linear equations in two variables graphically

IMPORTANT

Example 1:
Translate the followings words or phrases into mathematical language.

a) 7 times as many years increased by 3

$7y + 3$

b) 3 less than a number → $x - 3$
3 decreased by a number

$3 - x$

c) Option A charges a one-time $30 fee and then $8 per hour → $30 + 8h$
Option B charges $14 per hour

$14h$

avoid using the alphabet letter "o" as it looks like zero

Example 2: During a performance by a theatre company, the main act was on stage for 3 min less than twice the time of the opening act. Together, the two acts performed for 132 min.

a) Write a system of linear equations to represent the length of time each act performed.

m = the main act

a = the opening act

① assign variables to what you're comparing

② $m = 2a - 3$
$m + a = 132.$

$$\begin{pmatrix} m = y \\ a = x \end{pmatrix}$$

~~Review~~ Finding point of intersection on calc

b) What is the solution to their linear system? What does the solution represent?

$m = 2a - 3$

$m + a = 132 \rightarrow m = 132 - a$

(45, 87) → meaning the opening act is on stage for 45 mins, and the main act is on stage for 87 mins

① enter equations
② change window
③ Find intersect
↳ 2^{nd} → Trace
↳ 5: Intersect
↳ Enter 3 times

Example 3: Two pools start draining at the same time. The larger pool contains 54 675 L of water and rains at a rate of 25L/min. The smaller pool contains 35 400 L of water and drains at a rate of 10L/min.

a) Model the draining of the pools using a system of linear equations.

$P_1: L = -25m + 54675$

$P_2: L = -10m + 35400$

b) Represent the linear system graphically on your calculator. Describe how the information shown in the graph relates to the pools.

Point of intersection (1285, 22550)

Means, when both pools drain, they both have 22550L after 1285 mins.

Example 4: Jamie is travelling with her family from Castlegar, BC, to Pincher Creek, AB. Her dad and cousin do all of the driving. The 440-km trip takes 5.25 hours, excluding stops. Jamie's father drives at an average speed of 90 km/hr. Her cousin drives at 80km/hr.

a) What system of linear equations could help Jamie determine the length of time each person drove?

$d = 440 - 90t_f$

$d = 440 - 80t_c$

b) How many hours does each person drive? $d = 0$

$$0 = 440 - 90t$$
$$-440 \quad -440$$
$$\frac{-440}{-90} = \frac{-90t}{-90}$$
$$\boxed{4.9 \text{ hrs} = t_f}$$

$$0 = 440 - 80t$$
$$-440 \quad -440$$
$$\frac{-440}{-80} = \frac{-80t}{-80}$$
$$\boxed{5.5 \text{ hrs} = t_c}$$

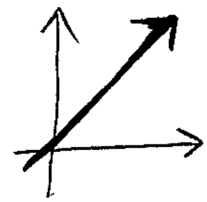
Key Ideas

- When modelling word problems, assign variables that are meaningful to the context of the problem.
- To assist in visualizing or organizing word problem, you can use a diagram and/or a table of values
- If a situation involves quantities that change at constant rates, you can represent it using a system of linear equations.
- If you know the initial value and rates, you can write the equations directly in slope-intercept form because the initial value is the y-intercept and the rate is the slope. Otherwise, you can determine the rate of change using start and end values.

Textbook Questions: Pg. 440 #1- 8, 10-11, 13

8.3 Number of Solutions for Systems of Linear Equations

Outcome: 1. Interpret graphical reasoning through the study of relations
9. Solve problems that involve systems of linear equations in two variables graphically



Definitions:
Coincident Lines: lines that occupy the same position.
- In a graph of two coincident lines, any point of either line lies on the other line.

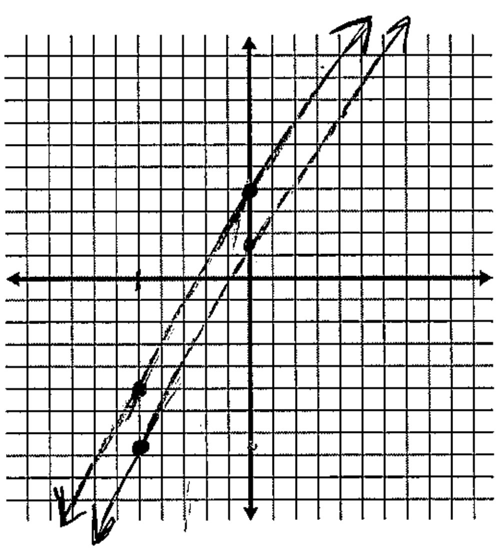
Example 1:
Four runners travel on a long, straight stretch of the Trans-Canada Highway. Their current distances and speeds are shown in the table of values.

	Current Distance (km)	Current Speed (km/h)
Sarah	4	9
Steve	2.5	9
Lucy	3	11
Mark	4	9

$d = 9t + 4$
$d = 9t + 2.5$
$d = 11t + 3$
$d = 9t + 4$

For each pair of vehicles below,
- Write a system of linear equations representing their travel from this point forward
- Graph each system of linear equations
- Identify and interpret the solution to each linear system

a) Sarah and Steve

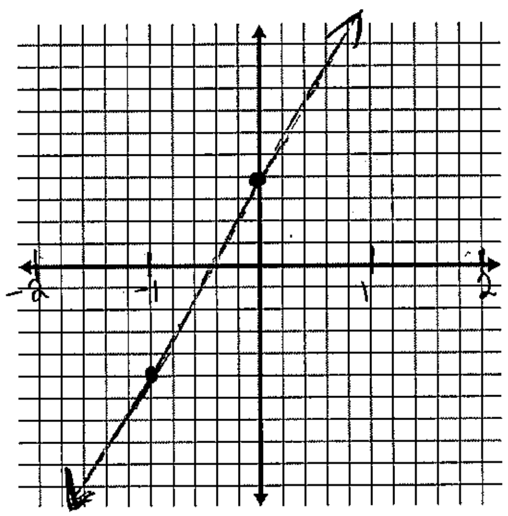


Since,
- the lines are parallel
  (same slope, different y-inter.)
- don't have an intersection point

There are no solutions

b) Sarah and Mark

$d = 9t + 4$
$d = 9t + 4$

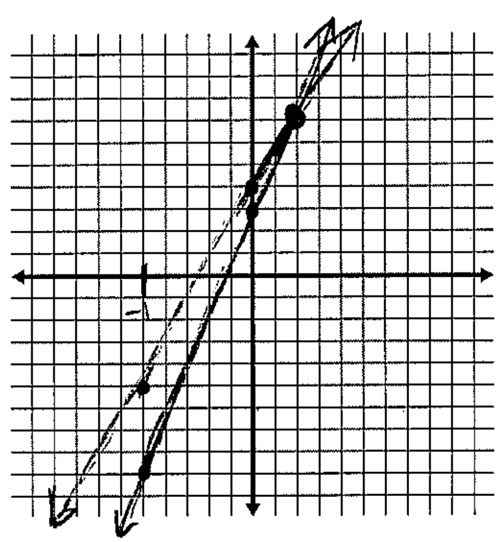


Since,
- they are the exact line equation (coincident lines)
- have infinite points of intersection

They have <u>infinite solutions</u>

c) Lucy and Mark

$d = 11t + 3$
$d = 9t + 4$



Since
- they have a different slope, and y-intercept
- have one point of intersection

They have <u>one solution</u>

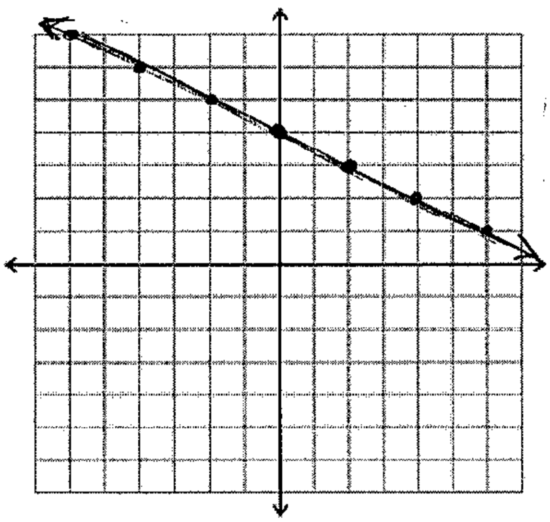
Example 2:

Predict the number of solutions for each system of linear equations. Explain your reasoning, and then confirm each answer by graphing the linear system.

a) $x + 2y = 4 \rightarrow y = -\frac{1}{2}x + 4$

$y = -\frac{1}{2}x + 4$

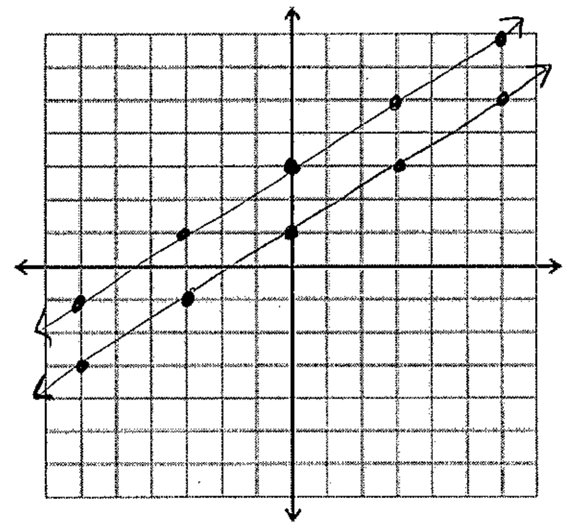
• same equation, thus infinite solutions



* b) $6y - 4x = 6 \rightarrow y = \frac{2}{3}x + 1$

$y = \frac{2}{3}x + 3$

• have the same slope, but different y-intercepts (parallel) therefore, no solutions

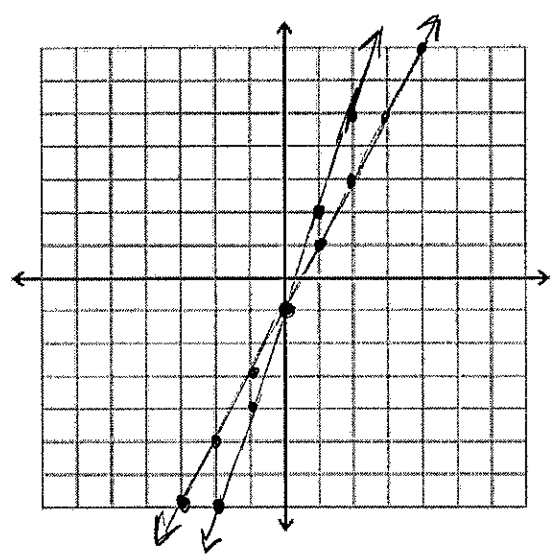


c) $y = 3x - 1$

$y = 2x - 1$

have the same y-intercept, but different slopes.

Thus one solution at (0, -1)



Example 3:

Determine, by inspection, whether each linear system has an infinite number of solutions or no solution. Explain your reasoning.

a) $2x + 10y - 16 = 0$
$x + 5y - 8 = 0$

① Rearrange into slope-intercept form

$$2x + 10y - 16 = 0$$
$$-2x \quad +16 \quad -2x + 16$$

$$\frac{10y}{10} = \frac{-2x}{10} + \frac{16}{10}$$

$$\boxed{y = -\frac{1}{5}x + \frac{8}{5}}$$

$$x + 5y - 8 = 0$$
$$-x \quad +8 \quad -x + 8$$

$$\frac{5y}{5} = \frac{-x}{5} + \frac{8}{5}$$

$$\boxed{y = -\frac{1}{5}x + \frac{8}{5}}$$

② Analyze your equations

Exact same equation, therefore has an infinite number of solutions.

b) $x + 2y + 4 = 0$
$x + 2y - 6 = 0$

① Rearrange

$$x + 2y + 4 = 0$$
$$-x \quad -4 \quad -x - 4$$

$$\frac{2y}{2} = \frac{-x}{2} - \frac{4}{2}$$

$$\boxed{y = -\frac{1}{2}x - 2}$$

$$x + 2y - 6 = 0$$
$$-x \quad +6 \quad -x + 6$$

$$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$$

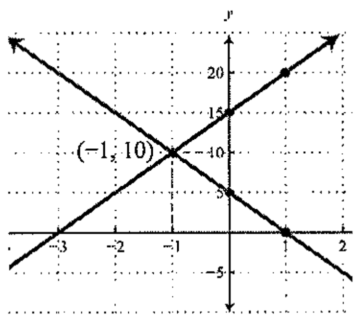
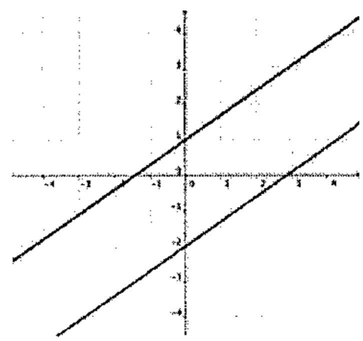
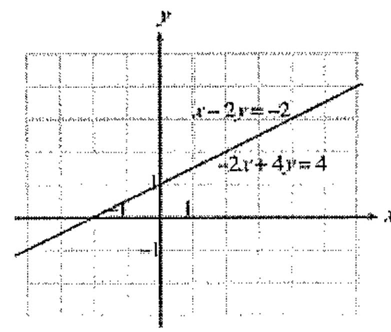
$$\boxed{y = -\frac{1}{2}x + 3}$$

② Analyze

They have the same slope, but different y-inter. (parallel lines), therefore, there are no solutions

Key Ideas

- A system of linear equations can have one solution, no solutions, or an infinite number of solutions.
- Before solving, you can predict the number of solutions for a linear system by comparing the slopes and y-intercepts of the equations.

Intersecting Lines	**Parallel Lines**	**Coincident Lines**
One solution 	No solutions 	Infinite solutions 
Different slopes	Same slope	Same slope
Y-intercepts can be the same or different	Different y-intercepts	Same y-intercept

- For some linear systems, reducing the equations to lowest terms and comparing the coefficients of the x-terms, y-terms, and constants may help you predict the number of solutions.

Textbook Questions: Pg. 454 # 1-9, 11-15