Math 10C

Name: Key

Chapter 4: Exponents and Radicals

4.1 Square Roots and Cube Roots

Review

1. Evaluate the following.

a. $\sqrt{81} = \sqrt{9^2} = 9$

b. $\sqrt{36} = \sqrt{6^2} = 6$

Outcome: Demonstrate an understanding of factors of whole numbers by determining the:

- Prime factors
- Greatest common factor
- Least common multiple
- Square root
- Cube root

Definitions:

Perfect Square: A number that can be expressed as the product of two equal factors

multiplying

the numbers being multiplied to get a number ($2 \times 3 = 6$)

Example:
- $16 = (4)(4)$ or 4^2
- $25 = (5)(5)$ or 5^2
- $36 = (6)(6)$ or 6^2

Perfect Squares

1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

Square Root: one of two equal factors of a number

Example:
- $\sqrt{49} = \sqrt{(7)(7)} = 7$

* can't take the square root of a negative number. *

Cube roots

1	1
2	8
3	27
4	64
5	125
6	216

Perfect Cube: A number that is the product of three equal factors

Example:
- $64 = (4)(4)(4) = 4^3$
- $27 = (3)(3)(3) = 3^3$

Cube Root: one of three equal factors of a number

Example:
- $\sqrt[3]{512} = \sqrt[3]{(8)(8)(8)} = 8$
- $\sqrt[3]{125} = \sqrt[3]{(5)(5)(5)} = 5$

MATH → 4: $\sqrt[3]{(}$ → Enter

Prime Factorization: the process of writing a number written as a product of its prime factors.

Example: The prime factorization of 24 is 2 x 2 x 2 x 3.

$24 \div 2 = 12 \div 2 = 6 \div 2 = 3$

Tree Method:

24

2 × 12

2 × 6

2 × 3

Relationship between Square Roots and Perfect Squares (Cube roots and Perfect Cubes)

The number 16 is a perfect square. It is formed by multiplying the same number, 4, twice together.

$(4)(4) = 4^2 = 16$

The square root of 16 is 4. $\sqrt{16} = \sqrt{(4)(4)} = \sqrt{4^2} = 4$

This is the same for perfect cubes and cube roots, however, the only difference is you the perfect cube is formed by multiplying the same number three times together.

Example 1: Identifying Perfect Squares and Perfect Cubes

State whether each of the following numbers is a perfect square or a perfect cube, both, or neither.

a) 144

Method 1: Tree Method

Can't group into 3's, so NOT perfect cube

144 → 2, 72; 72 → 2, 36; 36 → 2, 18; 18 → 2, 9; 9 → 3, 3

$2 \times 2 \times 3$
$= 12$

$\therefore$ perfect square

Method 2: Factor out perfect squares/cubes

$\sqrt{144} = \sqrt{4 \cdot 36}$
$= \sqrt{4} \cdot \sqrt{36}$
$= 2 \cdot 6$
$= 12$

x	x^2
1	1
2	$(2)^2 = 4$
3	$(3)^2 = 9$
4	$(4)^2 = 16$
5	$(5)^2 = 25$

b) 512

Method 1: Tree Method

512 → 2, 256; 256 → 2, 128; 128 → 2, 64; 64 → 2, 32; 32 → 2, 16; 16 → 2, 8; 8 → 2, 4; 4 → 2, 2

Can't group them into groups of 2, so NOT a perfect square

$\therefore$ perfect cube

Method 2: Factor out perfect square/cubes

$\sqrt[3]{512} = \sqrt[3]{8 \cdot 64}$
$= \sqrt[3]{8} \cdot \sqrt[3]{64}$
$= 2 \cdot 4$
$= 8$ ✓

$\sqrt{512} = \sqrt{16 \cdot 32}$
$= \sqrt{16} \cdot \sqrt{32}$
$= 4 \cdot \sqrt{32}$
$= 4 \cdot \sqrt{4 \cdot 8}$
$= 4 \cdot 2\sqrt{8}$
$= 4 \cdot 2 \cdot 2\sqrt{2}$ ✗

x	x^3
1	1
2	8
3	27
4	64
5	125

c) 356

Method 3: Calculator

$\sqrt{356} = 18.8796...$

$\sqrt[3]{356} = 7.08...$

Neither

Example 2:

Evaluate the following.

a) $\sqrt{64}$

$\sqrt{64} = \sqrt{4 \cdot 16}$

$= \sqrt{4} \cdot \sqrt{16}$

$= 2 \cdot 4 = \boxed{8}$

b) $\sqrt{144x^2}$

$\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$

$= \sqrt{144} \cdot x$

$= \sqrt{4 \cdot 36} \cdot x$

$= \sqrt{4} \cdot \sqrt{36} \cdot x$

$= 2 \cdot 6 \cdot x = \boxed{12x}$

c) $\sqrt[3]{125}$

$\sqrt[3]{125} = 5$

d) $\sqrt[3]{27a^3}$

$\sqrt[3]{27a^3}$

$= \sqrt[3]{27} \cdot \sqrt[3]{a^3}$

$= 3 \cdot a$

$= 3a$

x	x^3
1	1
2	8
3	27
4	64
5	125

Example 3:

A floor mat for gymnastics is a square with an area of 196m². What is its side length?

① What is the formula to find area of a square?

- Rectangle: $A = lw$
- Square: $A = s^2$

② $A = s^2$

$\sqrt{196m^2} = \sqrt{s^2}$

$\sqrt{196m^2} = s$

$\sqrt{4 \cdot 49 \cdot m^2} = s$

$\sqrt{4} \cdot \sqrt{49} \cdot \sqrt{m^2} = s$

$2 \cdot 7 \cdot m = s$

$\boxed{14m = s}$

$\sqrt{196m^2}$

196 = 2 · 98; 98 = 2 · 49; 49 = 7 · 7

$\sqrt{m^2}$: m · m → m

2, 7

$2 \cdot 7 \cdot m = 14m = s$

Example 4:

The volume of a cubic box is 27 000 in^3. Use two methods to determine it's dimensions.

① What is the formula for volume of a cube?

$V = s^3$

② $\sqrt[3]{27000 in^3} = \sqrt[3]{s^3}$

$\sqrt[3]{27000 in^3} = s$

$\sqrt[3]{27 \cdot 1000 \cdot in^3} = s$

$= \sqrt[3]{27} \cdot \sqrt[3]{1000} \cdot \sqrt[3]{in^3}$

$= 3 \cdot \sqrt[3]{1000} \cdot in$

$= 3 \cdot \sqrt[3]{8 \cdot 125} \cdot in$

$= 3 \cdot \sqrt[3]{8} \cdot \sqrt[3]{125} \cdot in$

$= 3 \cdot 2 \cdot 5 \cdot in$

$\boxed{= 30 in}$

Key Ideas

- A perfect square is the product of two equal factors. One of these factors is called the square root.
  - 25 is a perfect square: $\sqrt{25} = 5$ because $5^2 = 25$
- A perfect cube is the product of three equal factors.
  One of these factors is called the cube root.
  - -216 is a perfect cube: $\sqrt[3]{-216} = -6$ because $(-6)^3 = -125$
- Some numbers will be BOTH a perfect square AND a perfect cube.
  - 15 625 is a perfect square : $125^2 = 15\ 625$
  - 15 625 is a perfect cube: $25^3 = 15\ 625$
- You can use diagrams or manipulatives, factor trees, or a calculator to solve problems involving square roots and cube roots.

Textbook Questions: Pg.158-159 #1-4, 6, 9, 10

Can you evaluate the following?

a) $\sqrt{-16}$

b) $\sqrt[3]{-27}$

4.2 Integral Exponents

exponent

Review:

1. Identify the base and the exponent, then evaluate the power. $3^2 = (3)(3) = 9$

base

2. Simplify and evaluate the following. (*hint: you'll need to use exponent laws)*
   a. $4^2 \times 4^5 =$ $4^2 \times 4^5 = 4^{2+5} = \boxed{4^7}$

   b. $(5^2)(5^3) =$ $(5)^{2+3} = \boxed{5^5}$

   c. $(3^2)^7 =$ $3^{2\times7} = \boxed{3^{14}}$

   d. $\frac{6^8}{6^5} =$ $6^{8-5} = \boxed{6^3}$

Outcome: Demonstrate an understanding of powers with integral exponents.

What is an integral exponent?

Integer number: is a whole number that can be either positive, negative, or zero.

Example: $(\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots)$

Exponent: determines how many times to multiply a number. Usually to the right and above the base.

Example: x^2 , 3^4

Combine the two…

Integral Exponent: the exponent of a number is either a positive or negative whole number.

Examples: (5^{-2}), 2^{-3}

!*!*!*REMEMBER THE EXPONENT LAWS*!*!*!

Exponent Law	Example
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$(3^{-2})(3^4) = 3^{-2+4} = 3^2$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, \ a \neq 0$	$\frac{x^3}{x^{-5}} = (x^{3-(-5)}) = x^8$
Power of a Power $(a^m)^n = a^{mn}$	$(0.75^4)^{-2} = 0.75^{(4)(-2)} = 0.75^{-8} \ or \ \frac{1}{0.75^8}$
Power of a Product $(ab)^m = (a^m)(b^m)$	$(4z)^{-3} = \frac{1}{(4z)^3} \ or \ \frac{1}{64z^3}$
Power of a Quotient $(\frac{a}{b})^n = \frac{a^n}{b^n}, \ b \neq 0$	$(\frac{t}{3})^{-2} = (\frac{3}{t})^2 = \frac{3^2}{t^2} = \frac{9}{t^2}$
Zero Exponent $a^0 = 1, \ a \neq 0$	$(4y^2)^0 = 1$ $\quad -(4y^2)^0 = -1$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}, \ a \neq 0$	$4^{-3} = \frac{1}{4^3}$
Quotient of Negative Power $\frac{1}{a^{-n}} = a^n, \ a \neq 0$	$\frac{1}{3^{-2}} = 3^2$

Example 1:

Write each product or quotient as a power with a single exponent.

a) $(7^6)(7^{-2})$

Use the exponent laws for multiplying or dividing powers with the same base.

Method 1: Add the exponents

$7^{6+(-2)} = 7^{6-2} = \boxed{7^4}$

Method 2: Use Positive Exponents

$7^6 \cdot \left(\frac{1}{7^2}\right) = \frac{7^6}{7^2} = 7^{6-2} = \boxed{7^4}$

b) $\frac{7^{-5}}{7^{3}}$

Method 1: Subtract the Exponents

$7^{-5-3} = 7^{-8}$

Method 2: Use Positive Exponents

$\left(\frac{1}{7^5}\right)\left(\frac{1}{7^3}\right) = \frac{1}{(7^5)(7^3)}$

$= \frac{1}{7^{5+3}} = \boxed{\frac{1}{7^8}}$

c) $\frac{(3y)^3}{(3y)^{-2}}$

Method 1: Subtract the Exponents

$(3y)^{3-(-2)} = (3y)^{3+2}$

$\boxed{=(3y)^5}$

Method 2: Use Positive Exponents

$\frac{(3y)^3}{\left(\frac{1}{(3y)^2}\right)} = (3y)^3(3y)^2 = (3y)^{3+2}$

$\boxed{=(3y)^5}$

d) $\frac{(-3.5)^4}{(-3.5)^{-3}}$

Method 1: Subtract the Exponents

$(-3.5)^{4-(-3)}$

$= (-3.5)^{4+3} \boxed{= (-3.5)^7}$

Method 2: Use Positive Exponents

$\frac{(-3.5)^4}{\left(\frac{1}{(-3.5)^3}\right)} = (-3.5)^4(-3.5)^3$

$= (-3.5)^{4+3}$

$\boxed{= (-3.5)^7}$

Example 2:

Simplify and evaluate where possible.

a) $[(0.6^3)(0.6^{-3})]^{-5}$

$[(0.6^3)(0.6^{-3})]^{-5} = [(0.6^3)^{-5}(0.6^{-3})^{-5}]$

$= (0.6^{-15})(0.6^{15})$

$= (0.6^{-15+15})$

$= (0.6^0) = \boxed{1}$

* Order of operations matter! *

1st distribute exponent into brackets

b) $\left(\frac{x^6}{x^{-4}}\right)^2$

$\frac{(x^6)^2}{(x^{-4})^2} = \frac{x^{12}}{x^{-8}} = x^{12-(-8)} = x^{12+8} = \boxed{x^{20}}$

Example 3:

Manitoba Agriculture, Food and Rural Initiatives staff conducted a grasshopper count. In one 25 km² area, there were 401 000 000 grasshoppers. Use the following table to assess the degree of grasshopper infestation in this area.

① $1\text{ km} = 1000\text{m}$

$$\left(\frac{1\text{ km}}{1000\text{ m}}\right)^2 = \frac{25\text{ km}^2}{?}$$

$$\frac{(1\text{km})^2}{(1000\text{ m})^2} = \frac{25\text{km}^2}{?}$$

$$\frac{1\text{km}^2}{1000000\text{m}^2} = \frac{25\text{ km}^2}{?}$$

25000000 m^2

② $\frac{401000000}{25000000}$

$= 16.04$

$\therefore$ severe.

Grasshopper Density
0 - 4 per square metre = very light
5 - 8 per square metre = light
9 - 12 per square metre = moderate
13 - 24 per square metre = severe
25 per square metre = very severe

Key Ideas

- A power with a negative exponent can be written as a power with a positive exponent.
  - Example:

$$2^{-5} = \frac{1}{2^5} \qquad \left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \left(\frac{4}{3}\right)^2$$

- You can use the exponent laws to simplify.

Exponent Laws	
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	Power of a Product $(ab)^m = (a^m)(b^m)$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$
Power of a Power $(a^m)^n = a^{mn}$	Zero Exponent $a^0 = 1$, $a \neq 0$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}$, $a \neq 0$	Quotient of Negative Power $\frac{1}{a^{-n}} = a^n$, $a \neq 0$

Textbook Questions: Pg.169-170 #1, 2, 3-6, 8, 10.

4.3 Rational Exponents

Review: Evaluate the following:

a) 4^3 $= (4)(4)(4) = 64$

b) 4^{-3} $= \frac{1}{(4)^3} = \frac{1}{(4)(4)(4)} = \frac{1}{64}$

Outcome: Demonstrate an understanding of powers with rational exponents.

Definition

<u>Rational Exponents:</u> the power of a number is in the form of a fraction.

Example: $4^{\frac{1}{3}}$, $x^{\frac{4}{3}}$

<u>Example 1:</u>

Write each expression as a power with a single exponent.

a) $(x^{1.5})(x^{3.5})$

$x^{1.5+3.5} = \boxed{x^5}$

b) $\frac{4^{\frac{1}{2}}}{4^{0.5}}$

$4^{0.5-0.5} = 4^0 = \boxed{1}$

Note: $4^{\frac{1}{2}} = 4^{0.5}$

c) $\frac{1.5^{\frac{4}{3}}}{1.5^{\frac{1}{6}}}$ $= 1.5^{\left(\frac{4}{3}-\frac{1}{6}\right)} = 1.5^{\left(\frac{8}{6}-\frac{1}{6}\right)} = \boxed{1.5^{\frac{7}{6}}}$

$\frac{2}{2}\left(\frac{4}{3}\right) - \frac{1}{6} = \frac{8}{6} - \frac{1}{6}$

<u>Example 2:</u>

Simplify and evaluate where possible.

a) $\left(27x^6\right)^{\frac{2}{3}}$ $= 27x^{\left(6 \times \frac{2}{3}\right)} = 27x^{\frac{12}{3}} = \boxed{27x^4}$

$\frac{6}{1} \times \frac{2}{3}$

b) $\left[\left(t^{\frac{4}{3}}\right)\left(t^{\frac{1}{3}}\right)\right]^9 = \left(t^{\frac{4}{3}}\right)^9\left(t^{\frac{1}{3}}\right)^9 = \left(t^{\frac{36}{3}}\right)\left(t^{\frac{9}{3}}\right)$

$= (t^{12})(t^{3})$

$= t^{12+3} = \boxed{t^{15}}$

Example 3:

Caylie invests \$5000 in a fund that increases in value at a rate of 12.6% per year. The bank provides a quarterly update on the value of the investment using the formula

$A = 5000(1.126)^{\frac{q}{4}}$, where q represents the number of quarterly periods and A represents the final amount of the investment.

a) What is the relationship between the interest rate of 12.6% and the value 1.126 in the formula?

interest rate: amount owing PLUS an extra cost for borrowing money.

So, 12.6% = extra cost, amount owing = 100%

100% + 12.6% = 112.6% $\boxed{= 1.126}$

b) What is the value of the investment after the 3rd quarter?

$q = 3$: $A = 5000(1.126)^{3/4}$

$A = 5000(1.0924)$

$A = \$5461.78$

Round!! Money is ALWAYS 2 decimal places.

c) What is the value of the investment after 3 years?

① How many quarters are in 3 years?

→ 4 quarters in 1 year

→ 4 × 3 = 12 quarters

② $q = 12$

$A = 5000(1.126)^{12/4}$

$A = 5000(1.126)^3$

$A = 5000(1.42763) \boxed{= \$7138.14}$

Key Ideas

- You can write a power with a negative exponent as a power with a positive exponent.
  - $(-9)^{-1.3} = \frac{1}{(-9)^{1.3}}$
- You can apply the above principle to the exponent laws for rational expressions

Exponent Laws	
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	Power of a Product $(ab)^m = (a^m)(b^m)$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	Power of a Quotient $(\frac{a}{b})^n = \frac{a^n}{b^n}$, $b \neq 0$
Power of a Power $(a^m)^n = a^{mn}$	Zero Exponent $a^0 = 1$, $a \neq 0$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}$, $a \neq 0$	Quotient of Negative Power $\frac{1}{a^{-n}} = a^n$, $a \neq 0$

- A power with a rational exponent can be written with the exponent in decimal or fractional form.
  - $x^{3/5} = x^{0.6}$

Textbook Questions: Pg. 180-182 #1, 3-6, 8, 10, 12.

Review: The Real Number System

Define each of the following terms below and fill in the graphic organizer to the right.

Natural Numbers:

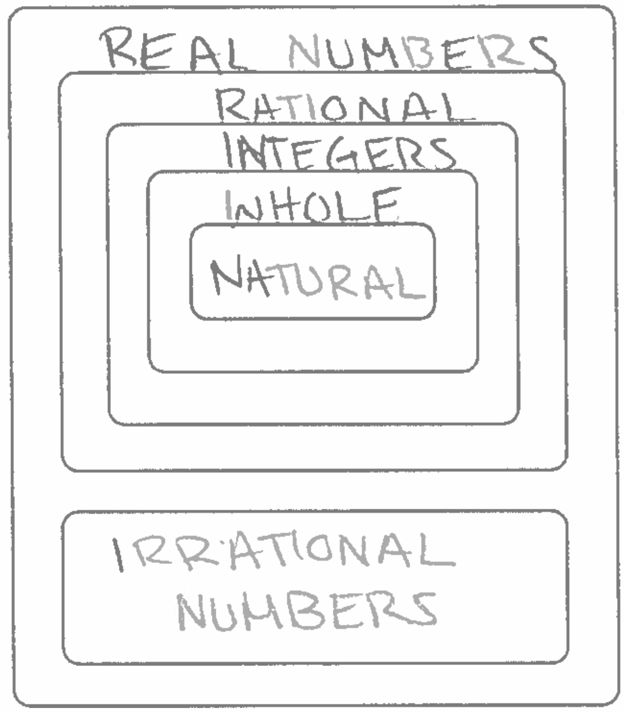
positive whole numbers excluding zero.

Whole Numbers:

A positive number with no decimal or fractions (0, 1, 2, 3, ...)

Integers:

A whole number either negative or positive (..., -3, -2, -1, 0, 1, 2...)



Rational Numbers:

any number that can be expressed as a fraction or quotient ($\frac{a}{b}$), where a and b are integers and not equal to 0.

Irrational Numbers:

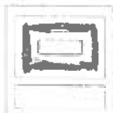
A number that CANNOT be expressed as a fraction. The decimal goes on forever, and doesn't repeat. ($\sqrt{2}$, π)

Real Numbers:

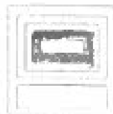
- Positive or negative, large or small, whole or decimal, are all real numbers
- Any rational or irrational number
- Can't be "imaginary" ie. ∞, $\sqrt{-1}$

Determine which sets each number belongs to. In the graphic organizer, shade in the sets.

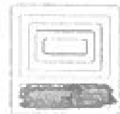
a) -4



b) 0

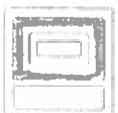


c) 1.273958...



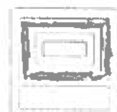
d) 7

e) $7.\overline{4}$



f) 4.93

g) $-\frac{2}{3}$



h) π

4.4 Irrational Numbers

Outcome: 1) Demonstrate an understanding of powers with rational and integral exponents.
2) Demonstrate an understanding of irrational numbers by:
- Representing, identifying, and simplifying irrational numbers
- Ordering irrational numbers

Example 1:

Identify the numbers as rational or irrational. You may use a calculator. Explain how you know.

$\frac{-5}{6}$

Rational

$-\frac{5}{6} = -0.83\overline{3}$

$\sqrt{11}$

irrational

$\sqrt{11} = 3.3166...$

$\sqrt[3]{\frac{8}{64}}$

rational

$\sqrt[3]{\frac{8}{64}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

Example 2:

Classify the following numbers as rational, irrational, or neither. You may use a calculator.

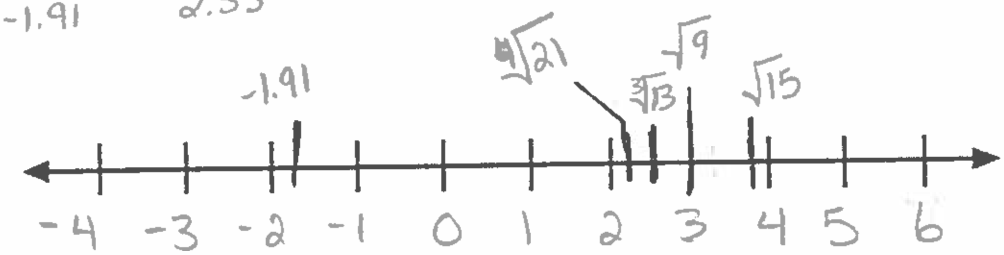
$\sqrt{7}$	0	$\sqrt[3]{125}$	$-\sqrt{2}$
irrational	rational	rational	irrational
$\frac{2}{3}$	$\sqrt{-2}$	$\sqrt{\frac{3}{5}}$	$\sqrt[3]{213}$
rational	neither	irrational	irrational
$\frac{6}{0}$	$\sqrt{81}$	8	$(27)^{\frac{1}{3}}$
neither	rational	rational	rational

Example 3:

Use a number line to order these numbers from least to greatest.

$\sqrt{15}$, $\sqrt{9}$, $\sqrt[3]{-7}$, $\sqrt[3]{13}$, $\sqrt[4]{21}$ — 2.1407

3.87 3 -1.91 2.35



Example 4:

Assume the Seabee Mine doubles its daily gold production to 360cm^3. What is the length of a cube of gold produced in a five-day period? (note the formula for volume, V, of a cube is $V = s^3$).

① 5 day period

$360cm^3 \times 5 = 1800\,cm^3$

② $V = s^3$

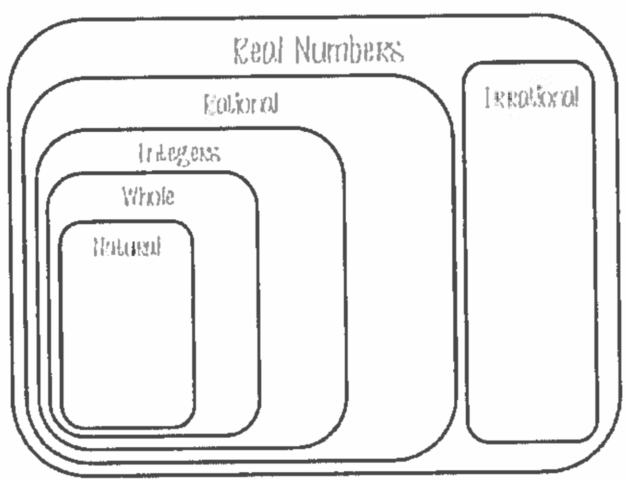
$\sqrt[3]{1800cm^3} = \sqrt[3]{s^3}$

$\sqrt[3]{1800\,cm^3} = s$

$12.1644\,cm = s$

Key Ideas

- Rational Numbers and irrational numbers form the set of real numbers.



- You can order radicals that are irrational numbers using a calculator to produce approximate values.

4.5 Mixed and Entire Radicals

Outcome: 1) Demonstrate an understanding of powers with rational and integral exponents.

2) Demonstrate an understanding of irrational numbers by:

- Representing, identifying, and simplifying irrational numbers
- Ordering irrational numbers

Definitions:

Radical: consists of a root symbol, an index, and a radicand

index $\rightarrow$ $\sqrt[n]{x}$ — root symbol

radicand

Radicand: the quantity under the radical sign

Index: indicates what root to take

A power can be expressed as a radical in the form:

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$$

ie. $3^{\frac{1}{3}} = \sqrt[3]{3^1} = \sqrt[3]{3}$

OR

$$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Example 1:

Express each power as an equivalent radical.

a) $10^{\frac{1}{4}} = \sqrt[4]{10^1} = \boxed{\sqrt[4]{10}}$

b) $1024^{\frac{1}{3}} = \sqrt[3]{(1024)^1} = \boxed{\sqrt[3]{1024}}$

c) $(x^4)^{\frac{3}{8}} = \sqrt[8]{(x^4)^3} = \sqrt[8]{x^{4\times3}} = \boxed{\sqrt[8]{x^{12}}}$

Example 2:

Express each radical as a power with a rational exponent.

a) $\sqrt[2]{125}$ $= 125^{\frac{1}{2}}$

b) $\sqrt[3]{y^5}$ $= y^{\frac{5}{3}}$

c) $\sqrt[n]{27^2}$ $= 27^{\frac{2}{n}}$

Definitions:

Mixed radical: the product of a rational number and a radical.

Example: $2\sqrt{5}$ *and* $\frac{1}{4}\sqrt[3]{7}$

Entire radical: the product of 1 and a radical.

Example: $\sqrt{45}$ *and* $\sqrt[3]{121}$

Example 3:

Identify whether the radical is a mixed radical or a entire radical.

a) $\sqrt{42}$ entire

b) $4\sqrt[3]{5}$ mixed

c) $3\sqrt{3}$ mixed

Example 4:

Express each mixed radical as an equivalent entire radical.

a) $9\sqrt[3]{4}$ $= \sqrt[3]{(9\times9\times9)\times4} = \sqrt[3]{729\cdot4} = \sqrt[3]{2916}$

b) $4.2\sqrt{18}$ $= \sqrt{(4.2\times4.2)\times18} = \sqrt{317.52}$

c) $\frac{1}{2}\sqrt{10}$ $= \sqrt{\left(\frac{1}{2}\times\frac{1}{2}\right)\times10} = \sqrt{\left(\frac{1}{4}\right)\times10} = \sqrt{2.5}$

Example 5:

Express each entire radical as an equivalent mixed radical.

a) $\sqrt{40}$ $= \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

b) $\sqrt{108}$ $= \sqrt{4 \cdot 27} = \sqrt{4} \cdot \sqrt{27} = 2\sqrt{27} = 2\sqrt{9 \cdot 3}$

$= 2\sqrt{9} \cdot \sqrt{3}$

$= 2 \cdot 3 \cdot \sqrt{3}$

$\boxed{= 6\sqrt{3}}$

c) $\sqrt[3]{32}$ $= \sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8} \cdot \sqrt[3]{4}$

$= 2\sqrt[3]{4}$

Key Ideas
• Radicals can be expressed as powers with fractional exponents. $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ The index of the radical has the same value as the denominator of the fractional exponent. $\sqrt[3]{10} = 10^{\frac{1}{3}}$ $\sqrt[3]{7^5} = 7^{\frac{5}{3}}$ • Radicals can be entire radicals such as $\sqrt{72}$ *and* $\sqrt[3]{96}$. They can also be mixed radicals such as $6\sqrt{2}$ *and* $2\sqrt[5]{3}$. You can convert between entire radicals and mixed radicals.

Textbook Questions: Pg. 192 #1-10