Math 10C Name: _______________________

Chapter 4: Exponents and Radicals

4.1 Square Roots and Cube Roots

Review

1. Evaluate the following.
   a. $\sqrt{81}$ b. $\sqrt{36}$

Outcome: Demonstrate an understanding of factors of whole numbers by determining the:

- Prime factors
- Greatest common factor
- Least common multiple
- Square root
- Cube root

Definitions:

Perfect Square: A number that can be expressed as the product of two equal factors

Example:

- $16 = (4)(4)$ or 4^2
- $25 = (5)(5)$ or 5^2
- $36 = (6)(6)$ or 6^2

Square Root: one of two equal factors of a number

Example:

- $\sqrt{49} = \sqrt{(7)(7)} = 7$

Perfect Cube: A number that is the product of three equal factors

Example:

- $64 = (4)(4)(4) = 4^3$
- $27 = (3)(3)(3) = 3^3$

Cube Root: one of three equal factors of a number

Example:

- $\sqrt[3]{512} = \sqrt[3]{(8)(8)(8)} = 8$
- $\sqrt[3]{125} = \sqrt[3]{(5)(5)(5)} = 5$

Prime Factorization: the process of writing a number written as a product of its prime factors.

Example: The prime factorization of 24 is 2 x 2 x 2 x 3. Tree Method:

$24 \div 2 = 12 \div 2 = 6 \div 2 = 3$

Relationship between Square Roots and Perfect Squares (Cube roots and Perfect Cubes)

The number ______ is a perfect square. It is formed by multiplying the same number, ____, twice together.

The square root of ____ is ____.

This is the same for perfect cubes and cube roots, however, the only difference is you the perfect cube is formed by multiplying the same number three times together.

Example 1: Identifying Perfect Squares and Perfect Cubes

State whether each of the following numbers is a perfect square or a perfect cube, both, or neither.

a) 144

Method 1: Tree Method	**Method 2: Factor out perfect squares/cubes**

b) 512

Method 1: Tree Method	**Method 2: Factor out perfect square/cubes**

c) 356

Method 3: Calculator

Example 2:

Evaluate the following.

a) $\sqrt{64}$

b) $\sqrt{144x^2}$

c) $\sqrt[3]{125}$

d) $\sqrt[3]{27a^3}$

Example 3:

A floor mat for gymnastics is a square with an area of 196m^2. What is its side length?

<u>Example 4:</u>

The volume of a cubic box is 27 000 in^3. Use two methods to determine it's dimensions.

Key Ideas

- A perfect square is the product of two equal factors. One of these factors is called the square root.
  - 25 is a perfect square: $\sqrt{25}$ = 5 because 5^2 = 25
- A perfect cube is the product of three equal factors.
  One of these factors is called the cube root.
  - -216 is a perfect cube: $\sqrt[3]{-216}$ = - 6 because $(- 6)^3$ = -125
- Some numbers will be BOTH a perfect square AND a perfect cube.
  - 15 625 is a perfect square : 125^2 = 15 625
  - 15 625 is a perfect cube: 25^3 = 15 625
- You can use diagrams or manipulatives, factor trees, or a calculator to solve problems involving square roots and cube roots.

Textbook Questions: Pg.158-159 #1-4, 6, 9, 10

4.2 Integral Exponents

Review:

1. Identify the base and the exponent, then evaluate the power. 3^2

2. Simplify and evaluate the following. (*hint: you'll need to use exponent laws)*
   a. $4^2 \times 4^5 =$

   b. $(5^2)(5^3) =$

   c. $(3^2)^7 =$

   d. $\frac{6^8}{6^5} =$

Outcome: Demonstrate an understanding of powers with integral exponents.

What is an integral exponent?

<u>Integer number:</u> is a whole number that can be either positive, negative, or zero.

Example:

<u>Exponent:</u> determines how many times to multiply a number. Usually to the right and above the base.

Example:

Combine the two…

<u>Integral Exponent:</u> the exponent of a number is either a positive or negative whole number.

Examples: (5^{-2}), 2^{-3}

!*!*!*REMEMBER THE EXPONENT LAWS*!*!*!

Exponent Law	Example
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$\left(3^{-2}\right)\left(3^4\right) = 3^{-2+4} = 3^2$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}\ ,\ a \neq 0$	$\frac{x^3}{x^{-5}} = \left(x^{3-(-5)}\right) = x^8$
Power of a Power $(a^m)^n = a^{mn}$	$\left(0.75^4\right)^{-2} = 0.75^{(4)(-2)} = 0.75^{-8}\ or\ \frac{1}{0.75^8}$
Power of a Product $(ab)^m = (a^m)(b^m)$	$(4z)^{-3} = \frac{1}{(4z)^3}\ or\ \frac{1}{64z^3}$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\ ,\ b \neq 0$	$\left(\frac{t}{3}\right)^{-2} = \left(\frac{3}{t}\right)^2 = \frac{3^2}{t^2} = \frac{9}{t^2}$
Zero Exponent $a^0 = 1,\ a \neq 0$	$\left(4y^2\right)^0 = 1$ $\quad -\left(4y^2\right)^0 = -1$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}\ ,\ a \neq 0$	$4^{-3} = \frac{1}{4^3}$
Quotient of Negative Power $\frac{1}{a^{-n}} = a^n,\ a \neq 0$	$\frac{1}{3^{-2}} = 3^2$

Example 1:

Write each product or quotient as a power with a single exponent.

a) $(7^6)(7^{-2})$

Use the exponent laws for multiplying or dividing powers with the same base.

Method 1: Add the exponents

Method 2: Use Positive Exponents

b) $\frac{7^{-5}}{7^3}$

Method 1: Subtract the Exponents

Method 2: Use Positive Exponents

c) $\frac{(3y)^3}{(3y)^{-2}}$

Method 1: Subtract the Exponents

Method 2: Use Positive Exponents

d) $\frac{(-3.5)^4}{(-3.5)^{-3}}$

Method 1: Subtract the Exponents

Method 2: Use Positive Exponents

Example 2:

Simplify and evaluate where possible.

a) $[(0.6^3)(0.6^{-3})]^{-5}$

b) $\left(\frac{x^6}{x^{-4}}\right)^2$

Example 3:

Manitoba Agriculture, Food and Rural Initiatives staff conducted a grasshopper count. In one 25 km^2 area, there were 401 000 000 grasshoppers. Use the following table to assess the degree of grasshopper infestation in this area.

Grasshopper Density
0 - 4 per square metre = very light
5 - 8 per square metre = light
9 - 12 per square metre = moderate
13 - 24 per square metre = severe
25 per square metre = very severe

Key Ideas

- A power with a negative exponent can be written as a power with a positive exponent.
  - Example:

$$2^{-5} = \frac{1}{2^5} \qquad \left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \left(\frac{4}{3}\right)^2$$

- You can use the exponent laws to simplify.

Exponent Laws	
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	Power of a Product $(ab)^m = (a^m)(b^m)$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$
Power of a Power $(a^m)^n = a^{mn}$	Zero Exponent $a^0 = 1,\ a \neq 0$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}$, $a \neq 0$	Quotient of Negative Power $\frac{1}{a^{-n}} = a^n,\ a \neq 0$

Textbook Questions: Pg.169-170 #1, 2, 3-6, 8, 10.

4.3 Rational Exponents

Review: Evaluate the following:

a) 4^3

b) 4^{-3}

Outcome: Demonstrate an understanding of powers with rational exponents.

Definition

<u>Rational Exponents:</u> the power of a number is in the form of a fraction.

Example: $4^{\frac{1}{3}}$, $x^{\frac{4}{3}}$

<u>Example 1:</u>

Write each expression as a power with a single exponent.

a) $(x^{1.5})(x^{3.5})$

b) $\frac{4^{\frac{1}{2}}}{4^{0.5}}$

c) $\frac{1.5^{\frac{4}{3}}}{1.5^{\frac{1}{6}}}$

<u>Example 2:</u>

Simplify and evaluate where possible.

a) $\left(27x^6\right)^{\frac{2}{3}}$

b) $\left[\left(t^{\frac{4}{3}}\right)\left(t^{\frac{1}{3}}\right)\right]^9$

Example 3:

Caylie invests $5000 in a fund that increases in value at a rate of 12.6% per year. The bank provides a quarterly update on the value of the investment using the formula $A = 5000(1.126)^{\frac{q}{4}}$, where q represents the number of quarterly periods and A represents the final amount of the investment.

a) What is the relationship between the interest rate of 12.6% and the value 1.126 in the formula?

b) What is the value of the investment after the 3rd quarter?

c) What is the value of the investment after 3 years?

Key Ideas

- You can write a power with a negative exponent as a power with a positive exponent.
  - $(-9)^{-1.3} = \frac{1}{(-9)^{1.3}}$
- You can apply the above principle to the exponent laws for rational expressions

Exponent Laws	
Note that *a* and *b* are rational or variable bases and *m* and *n* are the integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	Power of a Product $(ab)^m = (a^m)(b^m)$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$	Power of a Quotient $(\frac{a}{b})^n = \frac{a^n}{b^n}$, $b \neq 0$
Power of a Power $(a^m)^n = a^{mn}$	Zero Exponent $a^0 = 1,\ a \neq 0$
Power of a Negative Exponent $a^{-n} = \frac{1}{a^n}$, $a \neq 0$	Quotient of Negative Power $\frac{1}{a^{-n}} = a^n,\ a \neq 0$

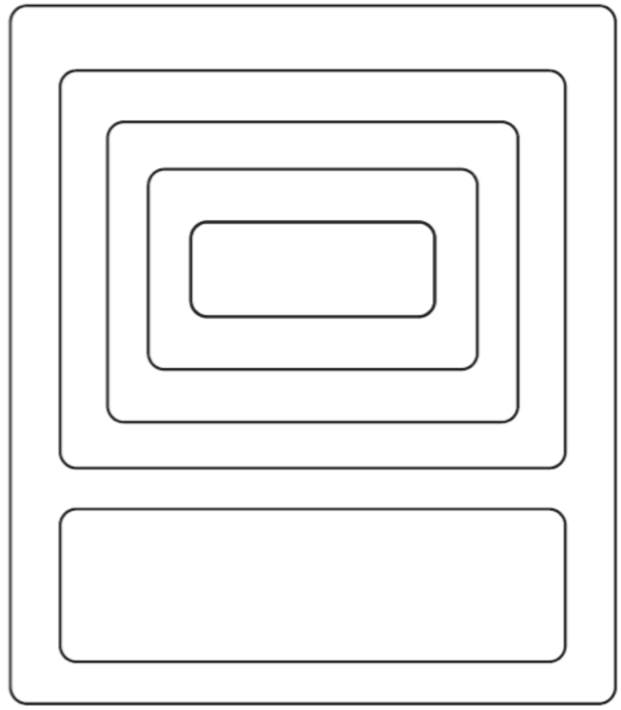
- A power with a rational exponent can be written with the exponent in decimal or fractional form.
  - $x^{3/5} = x^{0.6}$

Textbook Questions: Pg. 180-182 #1, 3-6, 8, 10, 12.

<u>Review: The Real Number System</u>

Define each of the following terms below and fill in the graphic organizer to the right.

Natural Numbers:



Whole Numbers:

Integers:

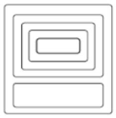
Rational Numbers:

Irrational Numbers:

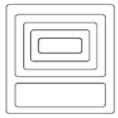
Real Numbers:

Determine which sets each number belongs to. In the graphic organizer, shade in the sets.

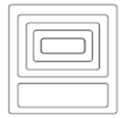
a) -4



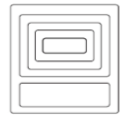
b) 0



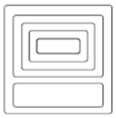
c) 1.273958...



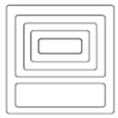
d) 7



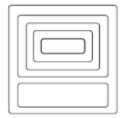
e) $7.\overline{4}$



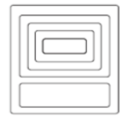
f) 4.93



g) $-\frac{2}{3}$



h) π



4.4 Irrational Numbers

Outcome: 1) Demonstrate an understanding of powers with rational and integral exponents.
2) Demonstrate an understanding of irrational numbers by:
- Representing, identifying, and simplifying irrational numbers
- Ordering irrational numbers

<u>Example 1:</u>

Identify the numbers as rational or irrational. You may use a calculator. Explain how you know.

$\frac{-5}{6}$ $\sqrt{11}$ $\sqrt[3]{\frac{8}{64}}$

<u>Example 2:</u>

Classify the following numbers as rational, irrational, or neither. You may use a calculator.

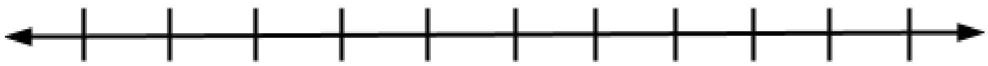
$\sqrt{7}$ 0 $\sqrt[3]{125}$ $-\sqrt{2}$

$\frac{2}{3}$ $\sqrt{-2}$ $\sqrt{\frac{3}{5}}$ $\sqrt[3]{213}$

$\frac{6}{0}$ $\sqrt{81}$ 8 $(27)^{\frac{1}{3}}$

<u>Example 3:</u>

Use a number line to order these numbers from least to greatest.
$\sqrt{15}$, $\sqrt{9}$, $\sqrt[3]{-7}$, $\sqrt[3]{13}$, $\sqrt[4]{21}$

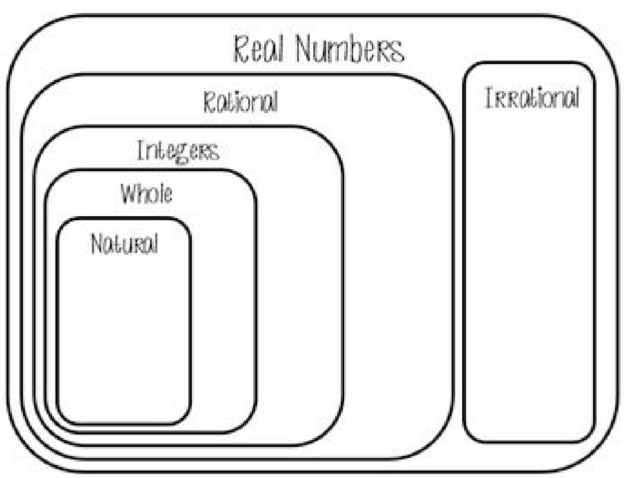


Example 4:

Assume the Seabee Mine doubles its daily gold production to $360 cm^3$. What is the length of a cube of gold produced in a five-day period? (note the formula for volume, V, of a cube is $V = s^3$).

Key Ideas

- Rational Numbers and irrational numbers form the set of real numbers.



- You can order radicals that are irrational numbers using a calculator to produce approximate values.

4.5 Mixed and Entire Radicals

Outcome: 1) Demonstrate an understanding of powers with rational and integral exponents.
2) Demonstrate an understanding of irrational numbers by:
- Representing, identifying, and simplifying irrational numbers
- Ordering irrational numbers

Definitions:

Radical: consists of a root symbol, an index, and a radicand

$$\sqrt[n]{x}$$

Radicand: the quantity under the radical sign

Index: indicates what root to take

A power can be expressed as a radical in the form:

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$$

OR

$$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Example 1:

Express each power as an equivalent radical.

a) $10^{\frac{1}{4}}$

b) $1024^{\frac{1}{3}}$

c) $(x^4)^{\frac{3}{8}}$

Example 2:

Express each radical as a power with a rational exponent.

a) $\sqrt{125}$

b) $\sqrt[3]{y^5}$

c) $\sqrt[n]{27^2}$

Definitions:

Mixed radical: the product of a rational number and a radical.

Example: $2\sqrt{5}$ *and* $\frac{1}{4}\sqrt[3]{7}$

Entire radical: the product of 1 and a radical.

Example: $\sqrt{45}$ *and* $\sqrt[3]{121}$

Example 3:

Identify whether the radical is a mixed radical or a entire radical.

a) $\sqrt{42}$

b) $4\sqrt[3]{5}$

c) $3\sqrt{3}$

Example 4:

Express each mixed radical as an equivalent entire radical.

a) $9\sqrt[3]{4}$

b) $4.2\sqrt{18}$

c) $\frac{1}{2}\sqrt{10}$

Example 5:

Express each entire radical as an equivalent mixed radical.

a) $\sqrt{40}$

b) $\sqrt{108}$

c) $\sqrt[3]{32}$

Key Ideas

- Radicals can be expressed as powers with fractional exponents.

  $$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

  The index of the radical has the same value as the denominator of the fractional exponent.

  $\sqrt[3]{10} = 10^{\frac{1}{3}}$ $\sqrt[3]{7^5} = 7^{\frac{3}{5}}$

- Radicals can be entire radicals such as $\sqrt{72}$ *and* $\sqrt[3]{96}$. They can also be mixed radicals such as $6\sqrt{2}$ *and* $2\sqrt[5]{3}$. You can convert between entire radicals and mixed radicals.

Textbook Questions: Pg. 192 #1-10