Name: Key

Chapter 3: Right Triangle Trigonometry 3.1 The Tangent Ratio

Outcome: Develop and apply the tangent ratio to solve problems that involve right triangles.

Definitions:

Adjacent side: the side that forms one of the arms of the acute angle

Hypotenuse: the side opposite to the right angle

Opposite side: the side across from the acute angle being considered

<u>Tangent Ratio</u>: for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side.





Example 1: Write the trigonometric ratio for tan A.

Solution



$tan \theta = \frac{Opposite}{Adjacent}$	Write the tangent
	ratio formula
$tan A = \frac{BC}{AC}$	Figure out what you are trying to find.
$tan A = \frac{14}{18}$	Substitute the lengths of the triangle sides
$tan A = \frac{14 \div 2}{18 \div 2}$	Finally, write the ratio into lowest terms.

tan A =

Example 2:

a) Calculate tan 36°. Round to 4 decimal places.

Solution

In your calculator input:



Answer: $tan 36^{\circ} = 0.7265$

b) Calculate $tan = \frac{5}{4}$. Round to the nearest tenth of a degree.

Solution

OR

Draw a right triangle below. Label your right angle.



Pick one of the two acute angles to be θ .

To complete the question we need to know what the trig ratio is for tan.

Now label the side opposite $\angle \theta$ as <u>5</u>. Label the side adjacent $\angle \theta$ as <u>4</u>.

Here we are trying to find an angle. Therefore, we need to use the inverse function ((*tan*⁻¹) on a calculator to apply the tangent ratio in reverse. *Note:* You can use the inverse function to find the size of an angle when 2 sides are known.



The angle θ is approximately <u>51.3</u>.

Example 3:

A ladder is leaning against a wall. It forms an angle of 63^{*} with the ground. The foot of the ladder is 2m from the wall. How far up does the ladder reach?

Solution

Draw a diagram and label with the given information



Label the sides in reference to the angle of 63° . Use the words *opposite* and *adjacent*. Label *x* on the side opposite the angle of 63° .

Write the trigonometric ratio.

$$tan \theta = \frac{Opposite}{Adjacent}$$

$$(tan \theta) \times adjacent = \left(\frac{Opposite}{Adjacent}\right) \times adjacent$$

$$(tan\theta) \times adjacent = opposite$$

 $(tan(63) \times a) = x$

<u>3.9m</u> = x

Manipulate the tangent ratio to solve for what you are looking for (here we are looking for the opposite).

Substitute values for angles and sides.

Solve by inputting it into your calculator.

OR

$$tan \theta = \frac{Opposite}{Adjacent}$$

$$tan(63^{\circ}) = \frac{x}{2m}$$
Substitute values for angles and sides.
$$2(tan \ 63^{\circ}) = 2\left(\frac{x}{2}\right)$$
Multiply both sides by 400 in order to get x by itself.

3. Determine the value of each variable. Express your answer to the nearest tenth of a unit.



4. The ramp is for wheelchair users. It has an incline of 6^{*}, at the highest point of the ramp is 3 ft. Determine the horizontal length, *x*, of the ramp. Express your answer to the nearest foot.

$$tan \theta = \frac{opposite}{adjacent}$$

$$(X) tan (6^{\circ}) = \frac{3ft}{x} (X)$$

$$(X) x tan (6^{\circ}) = \frac{3ft}{x} (X)$$

$$(X) x tan (6^{\circ}) = \frac{3ft}{tan (6^{\circ})}$$

$$X = \frac{3ft}{tan (6^{\circ})} = 28.543...ft$$

$$X = 29ft$$
Sentence: The length of the ramp is 29ft



$$2(\tan 63^\circ) = x$$
 Simplify

3.9m = x

Solve by inputting it into your calculator.

The height of the wall to the top of the ladder is approximately $\underline{\beta}.\underline{q}_{M}$ m.

Check Your Understandings:

 Mark the specified angle with a curved line. Label the hypotenuse, opposite, and adjacent sides to the angle. The first one has been started for you.

a) ∠X



b) ∠T



- 2. Determine each tangent ratio, to 4 decimal places.
 - a) $tan 74^{\circ} \approx 3.4874$
 - b) *tan* 45° ≈ _____
 - c) $\tan 37^{\circ} \approx 0.7536$
 - d) $tan 89^{\circ} \approx 57.39$

 A pilot approaches a runway at a constant angle of 3*. The end of the runway is 30 380 ft away. At what altitude should the aircraft be when beginning its decent? Express your answer to the nearest foot.

X 30 380 $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $(30380\text{F})_{\tan(3^{\circ})} = \frac{x}{30380\text{F}}$ $(30380\text{F})_{\tan(3^{\circ})} = \frac{x}{30380\text{F}}$ Solve for altitude. 30380 Ft x tan (3°) = x 1=1592.1483...Ft = X 1592ft = xSentence: The altitude of the aircraft is 1592 ft

Label the diagram with the information:

Textbook Questions: Pg.107-111 # 1, 3(a-c), 4(a,b), 5-7, 8, 11-13, & 15

3.2 The Sine and Cosine Ratios

Outcome: Develop and apply the sine and cosine ratio to solve problems that involve right triangles.

Definitions:

<u>Sine ratio</u>: for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse.

•
$$sin\theta = \frac{opposite}{hypotenuse}$$

<u>Cosine ratio</u>: for an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse.

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

<u>Primary Trigonometric Ratios:</u> the three ratios, sine, cosine, and tangent, defined in a right triangle.



Example 1

Write each trigonometric ratio using the triangle to the right.

a) sin L

b) cos L

m 1 8.

Example 2

a) Evaluate each sine ratio. $sin 60^{\circ} = 0.5660254...$

$$sin 30^\circ = 0, 5$$

Now round to 2 decimal places

sin 60° ≈ <u>0,87</u>

b) Evaluate each cosine ratio. $cos 45^* = 0.7070678...$

Now round to 2 decimal places $cos 45^* \approx 6.71$

sin 30° ≈ <u>0</u>. 5

 $cos 30^{\circ} = 0.8660254...$

cos 30°≈ 0.87

c) What is the measure of each angle, to the nearest degree? $sin \beta = 0.4384$ $\beta = sin^{-1}(0.4384)$ $\beta \approx \underline{26^{\circ}}$ $\theta \approx \underline{78^{\circ}}$

Example 3

A guy wire supporting a cell tower is 24 m long. The wire is attached at a height of 17 m up the tower. Determine the angle that the guy wire forms with the ground. Express your answer to the nearest degree.

Label the diagram with the known measurements. Let θ = unknown angle



You know the measurements of the hypotenuse and the <u>opposite</u> (adjacent side or hypotenuse) of the right triangle. So, use the <u>sine</u> ratio.

1) Write the trigonometric ratio

2) Substitute

$$\sin \theta = \frac{17m}{24m}$$

3) Use the inverse function

$$\Theta = \sin^{-1}\left(\frac{17m}{24m}\right)$$

4) Solve.

Check Your Understandings

- 1. Determine each trigonometric ratio, round to 4 decimal places.
 - a. cos (25°) ≈ <u>0.9063</u>
 - b. cos (72.3°) ≈ <u>0.3040</u>

c.	<i>sin</i> (60°) ≈_	0.8660
d.	sin (4.5°) \approx	0.0785

2. Write each trigonometric ratio in lowest terms.

a. sin A
Sin E = Opposite
hypotenuse
Sin A = 14
$$\left(\frac{3}{2}\right)$$

16 $\left(\frac{3}{2}\right)$
Sin A = 7
8



b. cos A

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

 $\cos A = \frac{8}{16}$
 $\cos A = \frac{1}{2}$

3. Calculate the measure of each angle, to the nearest degree.





Determine the value of *x*.
 Express your answer to the nearest tenth of a unit.

Known side: <u>hypotenuse</u> (adjacent or opposite or hypotenuse)

Unknown side: <u>adjacent</u> (adjacent or opposite or hypotenuse)

COSO = adjacent hypotenuse

$$(7)_{\cos}(52^{\circ}) = \frac{x}{7}(2)$$

$$7 \times \cos(52^{\circ}) = X$$

4.3096... = X
 $14.3 = X$



- 5. A hay elevator moves bales of hay to the barn loft. The bottom of the elevator is 8.5m from the barn. The loft opening is 5.5m above the ground.
 - a. Draw and label the known measurements.





180 yd

1 12 Tee box

b. What distance does a bale of hay travel along the elevator? Express your answer to the nearest tenth of a metre.

(hint: you will have to use pythagorean theorem: $c^2 = a^2 + b^2$)

$$c^{2} = (5.5m)^{2} + (8.5m)^{2}$$

 $c^{2} = 30.25m + 72.25m$
 $\sqrt{c^{2}} = \sqrt{102.5m}$
 $c = 10.12...m$ $C = 10.1m$

c. What is the angle from the top of the hay elevator to the barn?

$$\sin \theta = \frac{\Theta}{H}$$

$$\sin \theta = \frac{8.5m}{10.1m}$$

$$G = \sin^{-1} \left(\frac{8.5m}{10.1m} \right)$$

$$\Theta = 57.3077.^{\circ} = 57^{\circ} \left(\frac{5.7}{57} \right)$$

$$\Theta = 57.3077.^{\circ} = 57^{\circ} \left(\frac{5.7}{57} \right)$$

$$G = 57.3077.^{\circ} = 57^{\circ} \left(\frac{5.7}{57} \right)$$

$$\Theta = 57^{\circ} \left(\frac{5.7}{57} \right)$$

6. At a winter festival, the organizers set up an ice golf track. The track is in the shape of a right angle. The distance from the tee box to the vertex of the right angle is 180 yds.

can use ANY trios ratio.

a. The angle from the tee box to the flag at the other end of the track is 34°. Draw and label the angle.

b. Determine the direct distance, to the nearest yard, from the tee box to the flag.

$$cos \theta = A$$

$$H$$

$$(X) cos (34^{\circ}) = \frac{180 \text{ yd}}{x} (X)$$

$$(X) \times cos (34^{\circ}) = \frac{180 \text{ yd}}{x} (X)$$

$$(X) \times cos (34^{\circ}) = \frac{180 \text{ yd}}{x} (X)$$

$$(X) \times cos (34^{\circ}) = 217.119...9 \text{ yd} (X = 217.42)$$

$$X = \frac{180 \text{ yd}}{cos (34^{\circ})} = 217.119...9 \text{ yd} (X = 217.42)$$
Sentence: The distance from the tee box to the flag is altryd.

c. Determine the distance, to the nearest yard, from the tee box to the flag

Find the distance from the vertex to the flag. We need to $a^2+b^2=c^2$ $b^2=d^2-a^2$ $b^2=d^2-a^2$ $b^2=d^2-b^2=14689$ $b^2=a^2-180^2$ $b^2=121.198...9$ Distance of fairway = 180 + distance from vertex to flag =1801.1468= 180+ 121 49

Sentence: The distance of the fairway is 301 yd

= 301 yd

d. How much shorter is the direct distance than the distance following the track.

Sentence: The direct distance is 84 ud sharter

Textbook Questions: Pg. 120 -124 # 1, 2(a,c-e), 3(a,b,e,f), 4b, 5, 6(a-c), 8, 11, 13, & 15.

3.3 Solving Right Triangles

Outcome: Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.



Easy Steps for Solving Word Problems

- 1. Identify the angle you are working with (angle of depression vs. angle of elevation)
- 2. Determine what the unknown is (OPP, ADJ, or the Angle?)
- 3. Which trig ratio do you need to use? Then manipulate your formula to solve for the unknown:

a.
$$Opp = Hyp \times sin\theta$$

b.
$$Hyp = \frac{Opp}{sin\theta}$$

c.
$$\theta = \sin^{-1} \left(\frac{Opp}{Hyp} \right)$$

- 4. Substitute the values you are given into the correct trigonometric ratio.
- 5. Put the values into your calculator and round your answer.
- 6. Write down your answer. Don't forget to round!
- 7. Complete the question with a statement.

Step 3 and 4 can be switched. Either way should give the same answer, so the method that works best for you.

Example 1

Alyssa wants to calculate the height of the First Nations Native Totem Pole. She positions the transit 24m to the side of the totem pole. If the height of Alyssa's transit is 1.7m, what is the height of the totem pole?

- a) On the diagram, label the known dimensions.
- b) Determine the height of the totem pole, to the nearest metre.

Solution

a) On the diagram, label the known dimensions.



b) Calculate AB: Write the trigonometric ratio $\tan \theta = \frac{\Theta}{A}$

Substitute and manipulate.

(24m)tan $(58^{\circ}) = \frac{x}{24m}$

Simplify.

$$dHm \times tan (58°) = X$$

Solve.

$$38.4080...m = X$$

Height of totem pole = height of transit + height from top of transit to top of totem pole.

$$H_{s} = 1.7m + 38m$$

$$H_{s} = 39.7m$$
Sentence: The height of the totem pole is 39.7m

Example 2

A balloonist flies over an empty football field. When the balloon is directly over 1 goal post, the angle of depression to the base of the other goal post is 53°. The distance between the 2 goals is 110 yds. Determine the height of the balloon. Express your answer to the nearest yard.





1. Write the trigonometric ratio.



2. Substitute for 0 and the opposite side

$$\tan(53^\circ) = \frac{h}{10yd}$$

3. Multiply both sides by 110yd.

$$(110 \text{ yd}) \tan (53^\circ) = \frac{h}{10 \text{ yd}} (110 \text{ yd})$$

4. Simplify and Solve.

$$110 \text{ yd} \times \tan(53^\circ) = h$$

 $145.9749...\text{ yd} = h$
 $146 \text{ yd} = x$

Sentence: The beight of the balloon is 146yd

Solve AFED Example_3 Solve the triangle. Express each measurement to the nearest whole unit. Find the measurements of \angle _ \bigcirc and \angle _ \vdash and the length of 42 m side FD . F D 31m Find ∠D: The known lengths are the adjacent side and the <u>Sposite</u> (hypotenuse or opposite). Use the <u>tangent</u> (sine or cosine or tangent) ratio. tan $\theta = \frac{\Theta}{A}$ Write the trigonometric ratio. $\tan \theta = \frac{42m}{31m}$ Substitute. $\theta = \tan^{-1}\left(\frac{42m}{31m}\right)$ Use the inverse function. A = 53,569 ... ° $A = 54^{\circ}$ *Find* ∠F: ∠F = 180° - (<u>90°</u> + <u>54°</u>) 2F = 180° - 1440 Solve for ∠F LF = 36° Find the length of FD:

Use the trigonometric ratio: Sine For the ratio, use $\angle \underline{F}$. The unknown side is the <u>hypotenuse</u>. $Sin (2 = \bigcirc_{H}^{2}$ $(H) sin (36^{\circ}) = 3lm (H)$ $H = 3lm (36^{\circ}) = 52m$ Sentence: The length of FD is 52m H = 52mH

Check Your Understandings

1. Solve the triangle, to the nearest tenth of a unit.



Let θ = the unknown angle measurement.



$$cos \theta = \frac{A}{H}$$
(10) $cos(30^{\circ}) = \frac{x}{10}$ (10)
10 $x cos(30^{\circ}) = x$
8,660254... = x
 $8,7 = x$

c. Solve for y:

$$\sin \theta = \frac{0}{H}$$

(10) $\sin (30^{\circ}) = \frac{4}{10}$ (10)
 $10x \sin (30^{\circ}) = 4$
 $5 = 4$

2. Calculate the length of BC, to the nearest tenth of a centimeter.



Step 1: Draw 2 separate right triangles to model the large one. Label the measurements.



3. Determine the measure of \angle EDF, to the nearest tenth of a degree.





3

Calculate ∠EDG:



Calculate ∠GDF:



Calculate \angle EDF by finding the total of the 2 angles:

$$LEDF = LEDG + LGDF$$

 $LEDF = 32.4^{\circ} + 32.2^{\circ}$
 $LEDF = 64.6^{\circ}$

4. Circle the type of angle that describes each angle in the diagram.

- a) ∠1: ANGLE OF DEPRESSION or ANGLE OF ELEVATION
- b) 22: ANGLE OF DEPRESSION of ANGLE OF ELEVATION
- c) ∠3: ANGLE OF DEPRESSION or ANGLE OF ELEVATION
- d) ∠4: ANGLE OF DEPRESSION or ANGLE OF ELEVATION

5. An airplane spots an air traffic control tower at an angle of depression of 52*. The airplane is 850m above the ground. What is the distance of the line of sight from the airplane to the control tower? Express your answer to the nearest metre.

Let $\underline{\checkmark}$ = the direct distance from the airplane to the tower.





Sentence: The direct distance between the control tower and the airplane is 1079m 6. A photographer takes a picture of a polar bear from a helicopter. The angle of depression from the helicopter is 58°. The height of the helicopter is 140m. How far away is the helicopter from the polar bear? Express your answer to the nearest hundredth of a metre.

Draw a diagram to model the problem. $\tan \theta = \frac{0}{A}$

