

Chapter 3: Right Triangle Trigonometry

3.1 The Tangent Ratio

Outcome: Develop and apply the tangent ratio to solve problems that involve right triangles.

Definitions:

Adjacent side: the side that forms one of the arms of the acute angle

Hypotenuse: the side opposite to the right angle

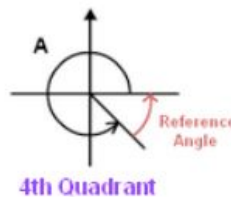
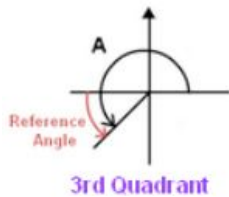
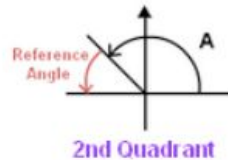
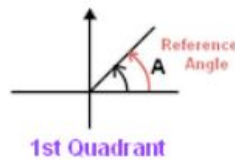
Opposite side: the side across from the acute angle being considered

Tangent Ratio: for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side.

$$\bullet \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Key Ideas

- In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal
- The sides of a right triangle are labeled according to a reference angle .
 - Reference angle is the angle between the x-axis and the terminal arm.



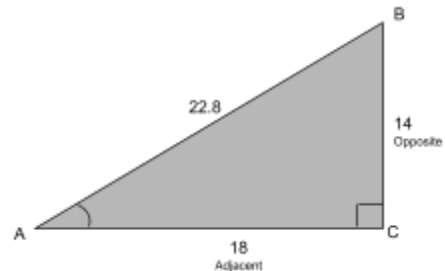
- The tangent ratio:
 - $\tan\theta = \frac{\text{length of side opposite to the reference angle}}{\text{length of side adjacent to the reference angle}}$
- The tangent ratio is used to:
 - Determine the measure of one of the acute angles when the lengths of both legs in a right triangle are known
 - Determine a side length if the measure of one acute angle and the length of one leg of a right triangle are known

Easy Steps for Solving Tangent Questions

1. Label the sides of the right triangle
2. Work out what you are trying to find. Is the unknown the OPP, ADJ, or the Angle?
3. Manipulate your formula to solve for the unknown:
 - a. $Opp = Adj \times \tan\theta$
 - b. $Adj = \frac{Opp}{\tan\theta}$
 - c. $\theta = \tan^{-1} \left(\frac{Opp}{Adj} \right)$
4. Substitute the values you are given into the equation.
5. Put the values into your calculator and round your answer.
 - a. *MAKE SURE YOUR CALCULATOR IS IN DEGREES MODE*
6. Write down your answer. Don't forget to round!

Note that step 3 and 4 can be switched. Either way should give the same answer, so the method that works best for you.

Example 1: Write the trigonometric ratio for $\tan A$.



Solution

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan A = \frac{14}{18}$$

$$\tan A = \frac{14 \div 2}{18 \div 2}$$

$$\tan A =$$

Write the tangent ratio formula

Figure out what you are trying to find.

Substitute the lengths of the triangle sides

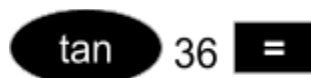
Finally, write the ratio into lowest terms.

Example 2:

- a) Calculate $\tan 36^\circ$. Round to 4 decimal places.

Solution

In your calculator input:



Answer: $\tan 36^\circ =$ _____

b) Calculate $\tan \theta = \frac{5}{4}$. Round to the nearest tenth of a degree.

Solution

Draw a right triangle below. Label your right angle.

Pick one of the two acute angles to be θ .

To complete the question we need to know what the trig ratio is for \tan .

$$\tan \theta =$$

Now label the side opposite $\angle \theta$ as _____.

Label the side adjacent $\angle \theta$ as _____.

Here we are trying to find an angle. Therefore, we need to use the inverse function (\tan^{-1}) on a calculator to apply the tangent ratio in reverse. *Note: You can use the inverse function to find the size of an angle when 2 sides are known.*

$$\tan \theta = \frac{5}{4}$$

$$\theta = \tan^{-1} \left(\frac{5}{4} \right) \quad \leftarrow \quad \text{2nd} \quad \text{tan} \quad (5 \div 4) \quad =$$

$$\theta = \underline{\hspace{2cm}}$$

OR

$$\tan \theta = \frac{5}{4}$$

$$\tan \theta = \underline{\hspace{2cm}} \quad \leftarrow \quad \text{What is } 5 \div 4 = ??$$

$$\theta = \underline{\hspace{2cm}} \quad \text{2nd} \quad \text{tan} \quad (1.25) \quad =$$

The angle θ is approximately _____.

Example 3:

A ladder is leaning against a wall. It forms an angle of 63° with the ground. The foot of the ladder is 2m from the wall. How far up does the ladder reach?

Solution

Draw a diagram and label with the given information

Let x = _____

Label the sides in reference to the angle of 63°. Use the words *opposite* and *adjacent*.

Label x on the side opposite the angle of 63°.

Write the trigonometric ratio.

$$\tan \theta = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

Manipulate the tangent ratio to solve for what you are looking for (here we are looking for the opposite).

$$(\tan \theta) \times \textit{adjacent} = \left(\frac{\textit{Opposite}}{\textit{Adjacent}} \right) \times \textit{adjacent}$$

$$(\tan \theta) \times \textit{adjacent} = \textit{opposite}$$

Substitute values for angles and sides.

$$\tan(63^\circ) \times 2 = x$$

Solve by inputting it into your calculator.

$$\underline{\hspace{2cm}} = x$$

OR

$$\tan \theta = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

$$\tan(63^\circ) = \frac{x}{2}$$

Substitute values for angles and sides.

$$2(\tan 63^\circ) = 2 \left(\frac{x}{2} \right)$$

Multiply both sides by 2 in order to get x by itself.

$$2(\tan 63^\circ) = x$$

Simplify

$$\underline{\hspace{2cm}} = x$$

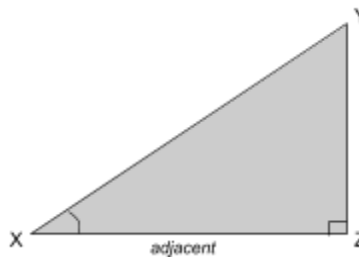
Solve by inputting it into your calculator.

The height of the wall to the top of the ladder is approximately _____ m.

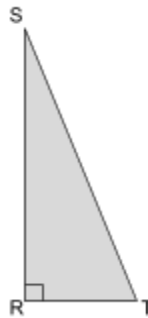
Check Your Understandings:

1. Mark the specified angle with a curved line.
Label the hypotenuse, opposite, and adjacent sides to the angle.
The first one has been started for you.

- a) $\angle X$



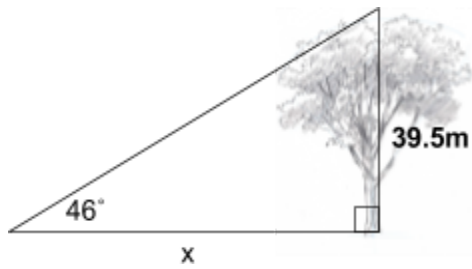
- b) $\angle T$



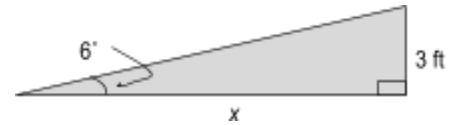
2. Determine each tangent ratio, to 4 decimal places.

- a) $\tan 74^\circ \approx \underline{\hspace{2cm}}$
- b) $\tan 45^\circ \approx \underline{\hspace{2cm}}$
- c) $\tan 37^\circ \approx \underline{\hspace{2cm}}$
- d) $\tan 89^\circ \approx \underline{\hspace{2cm}}$

3. Determine the value of each variable. Express your answer to the nearest tenth of a unit.



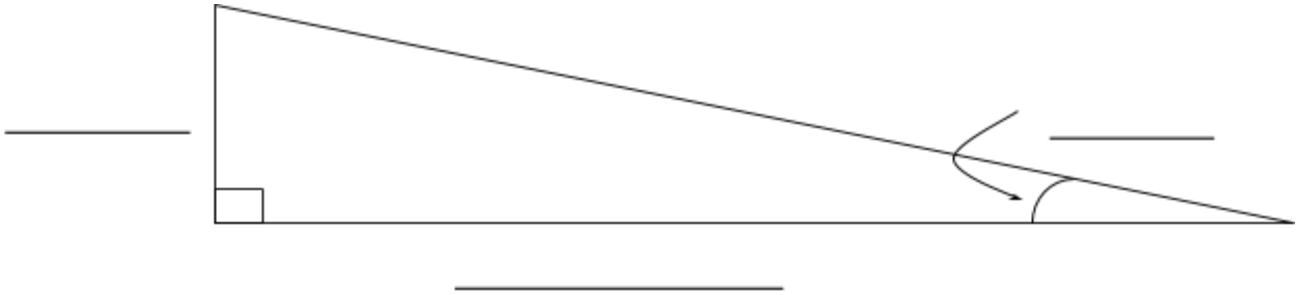
4. The ramp is for wheelchair users. It has an incline of 6° , at the highest point of the ramp is 3 ft. Determine the horizontal length, x , of the ramp. Express your answer to the nearest foot.



Sentence: _____

5. A pilot approaches a runway at a constant angle of 3° . The end of the runway is 30 380 ft away. At what altitude should the aircraft be when beginning its decent? Express your answer to the nearest foot.

Label the diagram with the information:



Solve for altitude.

Sentence: _____

Textbook Questions: Pg.107-111 # 1, 3(a-c), 4(a,b), 5-7, 8, 11-13, & 15

3.2 The Sine and Cosine Ratios

Outcome: Develop and apply the sine and cosine ratio to solve problems that involve right triangles.

Definitions:

Sine ratio: for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse.

- $\sin\theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$

Cosine ratio: for an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse.

- $\cos\theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$

Primary Trigonometric Ratios: the three ratios, sine, cosine, and tangent, defined in a right triangle.

Key Ideas	
<ul style="list-style-type: none"> • The sine ratio and cosine ratio compare the lengths of the legs of a right triangle to the hypotenuse. 	<ul style="list-style-type: none"> • Similar to the tangent ratio, sine and cosine ratios can be used to calculate side lengths and angle measures of right triangles.
$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$	$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$

Example 1

Write each trigonometric ratio using the triangle to the right.

a) $\sin L$

$$\sin L = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\sin L = \underline{\hspace{2cm}}$$

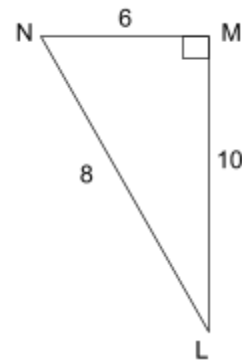
$$\sin L = \underline{\hspace{2cm}}$$

b) $\cos L$

$$\cos L = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\cos L = \underline{\hspace{2cm}}$$

$$\cos L = \underline{\hspace{2cm}}$$



Example 2

a) Evaluate each sine ratio.

$$\sin 60^\circ = \underline{\hspace{2cm}}$$

$$\sin 30^\circ = \underline{\hspace{2cm}}$$

Now round to 2 decimal places

$$\sin 60^\circ \approx \underline{\hspace{2cm}}$$

$$\sin 30^\circ \approx \underline{\hspace{2cm}}$$

b) Evaluate each cosine ratio.

$$\cos 45^\circ = \underline{\hspace{2cm}}$$

$$\cos 30^\circ = \underline{\hspace{2cm}}$$

Now round to 2 decimal places

$$\cos 45^\circ \approx \underline{\hspace{2cm}}$$

$$\cos 30^\circ \approx \underline{\hspace{2cm}}$$

c) What is the measure of each angle, to the nearest degree?

$$\sin \beta = 0.4384$$

$$\cos \theta = 0.2079$$

$$\beta = \sin^{-1}(0.4384)$$

$$\theta = \underline{\hspace{2cm}}$$

$$\beta \approx \underline{\hspace{2cm}}$$

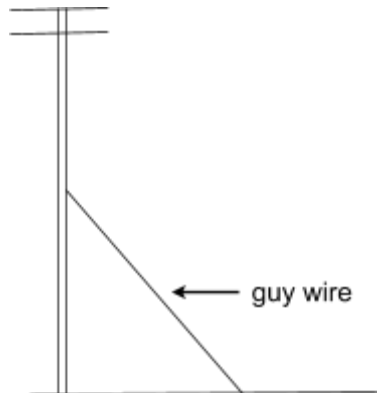
$$\theta \approx \underline{\hspace{2cm}}$$

Example 3

A guy wire supporting a cell tower is 24 m long. The wire is attached at a height of 17 m up the tower. Determine the angle that the guy wire forms with the ground. Express your answer to the nearest degree.

Label the diagram with the known measurements.

Let θ = unknown angle



You know the measurements of the hypotenuse and the _____ (*adjacent side or hypotenuse*) of the right triangle. So, use the _____ ratio.

1) Write the trigonometric ratio

2) Substitute

3) Use the inverse function

4) Solve.

Sentence: _____

Check Your Understandings

1. Determine each trigonometric ratio, round to 4 decimal places.

a. $\cos(25^\circ) \approx$ _____

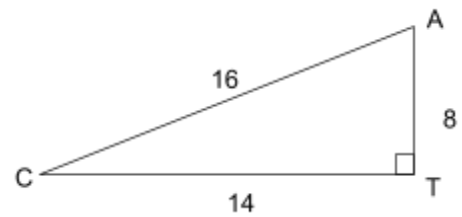
c. $\sin(60^\circ) \approx$ _____

b. $\cos(72.3^\circ) \approx$ _____

d. $\sin(4.5^\circ) \approx$ _____

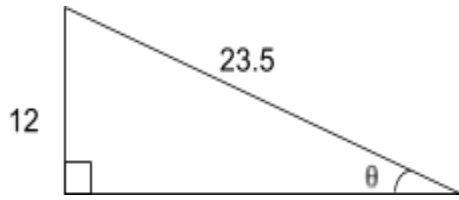
2. Write each trigonometric ratio in lowest terms.

a. $\sin A$



b. $\cos A$

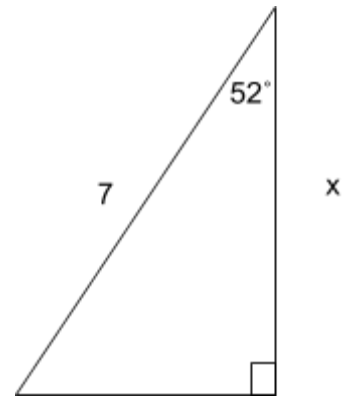
3. Calculate the measure of each angle, to the nearest degree.



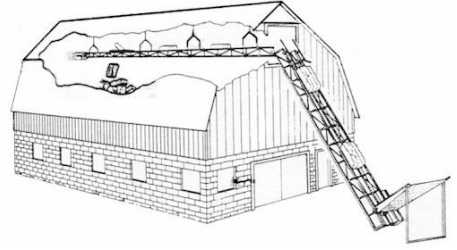
4. Determine the value of x .
Express your answer to the nearest tenth of a unit.

Known side: _____
(adjacent or opposite or hypotenuse)

Unknown side: _____
(adjacent or opposite or hypotenuse)



5. A hay elevator moves bales of hay to the barn loft. The bottom of the elevator is 8.5m from the barn. The loft opening is 5.5m above the ground.



- a. Draw and label the known measurements.

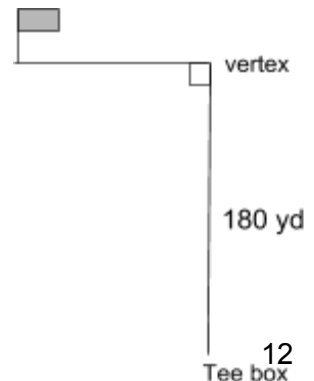
- b. What distance does a bale of hay travel along the elevator? Express your answer to the nearest tenth of a metre.

(*hint: you will have to use pythagorean theorem: $c^2 = a^2 + b^2$*)

- c. What is the angle from the top of the hay elevator to the barn?

6. At a winter festival, the organizers set up an ice golf track. The track is in the shape of a right angle. The distance from the tee box to the vertex of the right angle is 180 yds.

- a. The angle from the tee box to the flag at the other end of the track is 34° . Draw and label the angle.



- b. Determine the direct distance, to the nearest yard, from the tee box to the flag.

Sentence: _____

- c. Determine the distance, to the nearest yard, from the tee box to the flag if a ball follows the fairway.

Find the distance from the vertex to the flag.

Distance of fairway = 180 + distance from vertex to flag

Sentence: _____

- d. How much shorter is the direct distance than the distance following the track.

Sentence: _____

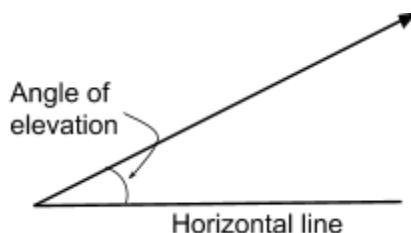
Textbook Questions: Pg. 120 -124 # 1, 2(a,c-e), 3(a,b,e,f), 4b, 5, 6(a-c), 8, 11, 13, & 15.

3.3 Solving Right Triangles

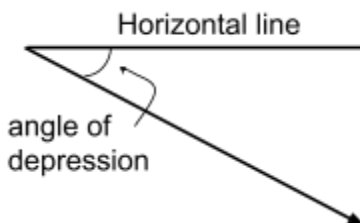
Outcome: Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Key Ideas

- **SOH-CAH-TOA**
- An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward.



- An angle of depression is the angle between the line of sight and the horizontal when the observer looks downward.



- To solve a triangle means to calculate all unknown angle measures and side lengths.

Easy Steps for Solving Word Problems

1. Identify the angle you are working with (*angle of depression vs. angle of elevation*)
2. Determine what the unknown is (OPP, ADJ, or the Angle?)
3. Which trig ratio do you need to use? Then manipulate your formula to solve for the unknown:
 - a. $Opp = Hyp \times \sin\theta$
 - b. $Hyp = \frac{Opp}{\sin\theta}$
 - c. $\theta = \sin^{-1}\left(\frac{Opp}{Hyp}\right)$
4. Substitute the values you are given into the correct trigonometric ratio.
5. Put the values into your calculator and round your answer.
6. Write down your answer. Don't forget to round!
7. Complete the question with a statement.

Step 3 and 4 can be switched. Either way should give the same answer, so the method that works best for you.

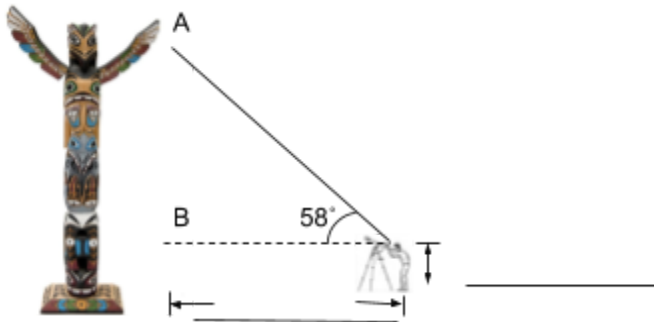
Example 1

Alyssa wants to calculate the height of the First Nations Native Totem Pole. She positions the transit 24m to the side of the totem pole. If the height of Alyssa's transit is 1.7m, what is the height of the totem pole?

- On the diagram, label the known dimensions.
- Determine the height of the totem pole, to the nearest metre.

Solution

- On the diagram, label the known dimensions.



- Calculate AB:

Write the trigonometric ratio

Substitute and manipulate.

Simplify.

Solve.

Height of totem pole = height of transit + height from top of transit to top of totem pole.

Sentence: _____

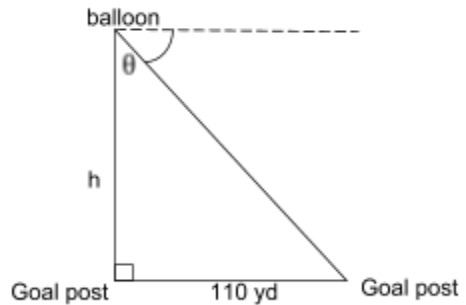
Example 2

A balloonist flies over an empty football field. When the balloon is directly over 1 goal post, the angle of depression to the base of the other goal post is 53° . The distance between the 2 goals is 110 yds. Determine the height of the balloon. Express your answer to the nearest yard.

Model your problem. Label the measure of the angle of depression.

Let $h =$ _____

Calculate θ .



1. *Write the trigonometric ratio.*
2. *Substitute for θ and the opposite side*
3. *Multiply both sides by 110yd.*
4. *Simplify and Solve.*

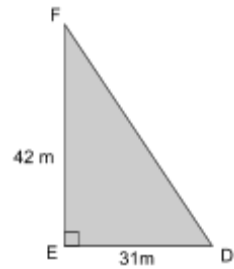
Sentence: _____

Example 3

Solve the triangle.

Express each measurement to the nearest whole unit.

Find the measurements of \angle _____ and \angle _____ and the length of side _____.



Find $\angle D$:

The known lengths are the adjacent side and the _____
(*hypotenuse or opposite*).

Use the _____ (*sine or cosine or tangent*) ratio.

Write the trigonometric ratio.

Substitute.

Use the inverse function.

Find $\angle F$:

$$\angle F = 180^\circ - (\text{_____} + \text{_____})$$

Solve for $\angle F$

Find the length of FD:

Use the trigonometric ratio:

Use the Pythagorean theorem:

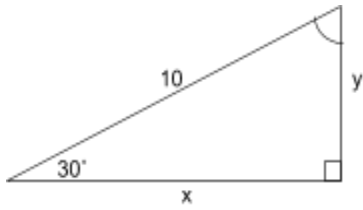
For the ratio, use \angle _____.

The unknown side is the _____.

Sentence: _____

Check Your Understandings

1. Solve the triangle, to the nearest tenth of a unit.



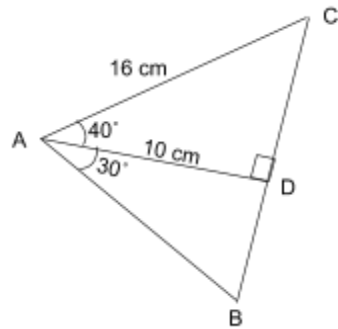
Let θ = the unknown angle measurement.

- a. Solve for θ .

- b. Solve for x :

- c. Solve for y :

2. Calculate the length of BC, to the nearest tenth of a centimeter.



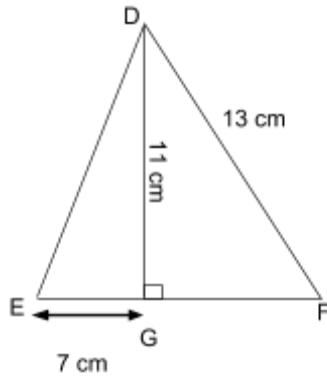
Step 1: Draw 2 separate right triangles to model the large one. Label the measurements.

Step 2: Calculate BD:

Step 3: Calculate DC:

Final step: Calculate BC:

3. Determine the measure of $\angle EDF$, to the nearest tenth of a degree.



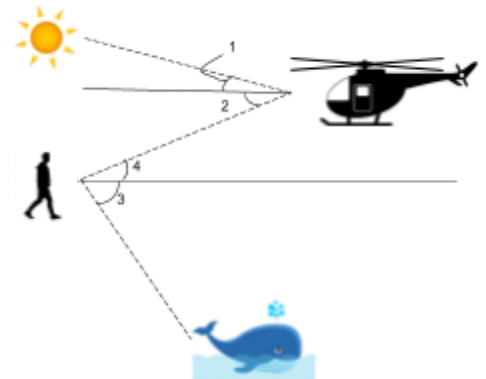
Calculate $\angle EDG$:

Calculate $\angle GDF$:

Calculate $\angle EDF$ by finding the total of the 2 angles:

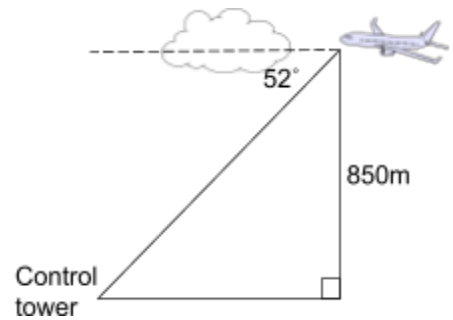
4. Circle the type of angle that describes each angle in the diagram.

- a) $\angle 1$: ANGLE OF DEPRESSION or ANGLE OF ELEVATION
- b) $\angle 2$: ANGLE OF DEPRESSION or ANGLE OF ELEVATION
- c) $\angle 3$: ANGLE OF DEPRESSION or ANGLE OF ELEVATION
- d) $\angle 4$: ANGLE OF DEPRESSION or ANGLE OF ELEVATION



5. An airplane spots an air traffic control tower at an angle of depression of 52° . The airplane is 850m above the ground. What is the distance of the line of sight from the airplane to the control tower? Express your answer to the nearest metre.

Let _____ = the direct distance from the airplane to the tower.



Sentence: _____

6. A photographer takes a picture of a polar bear from a helicopter. The angle of depression from the helicopter is 58° . The height of the helicopter is 140m. How far away is the helicopter from the polar bear? Express your answer to the nearest hundredth of a metre.
Draw a diagram to model the problem.

Sentence: _____

Textbook Questions: Pg.131 - 135 # 1(b-d), 2b, 3-4, 7, 8, 10, 12, & 14.